

[DAC61333] KALKULUS LANJUT

"Integral Lipat"

Semester Ganjil 2019-2020

Resmawan

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28 Oktober 2019

13.2. Integral Berulang

2.1 Menghitung Integral Lipat Dua sebagai Integral Berulang

Definition

Jika f terintegralkan pada persegi panjang $R = [a, b] \times [c, d]$, maka integral lipat dua dari f pada R dapat dihitung sebagai **integral berulang**:

$$\iint_R f(x, y) dA = \int_c^d \int_a^b f(x, y) dx dy$$

atau

$$\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx$$

Catatan: Pada cara pertama, ruang diiris sejajar sumbu x terlebih dahulu

2.2 Beberapa Contoh Soal

Examples

Hitunglah masing-masing integral berulang dari

$$1) \int_0^2 \int_0^3 (9 - x) \, dydx$$

$$2) \int_{-1}^1 \int_1^2 (x^2 + y^2) \, dx dy$$

$$3) \int_0^8 \int_0^4 \frac{64 - 8x + y^2}{16} \, dx dy$$

2.2 Beberapa Contoh Soal

Solution

$$\begin{aligned} 1) \int_0^2 \int_0^3 (9 - x) \, dy dx &= \int_0^2 (9y - xy) \Big|_0^3 dx \\ &= \int_0^2 [9y - xy]_0^3 dx \\ &= \int_0^2 [(9 \cdot 3 - x \cdot 3) - 0] dx \\ &= \int_0^2 [27 - 3x] dx \\ &= \left[27x - \frac{3}{2}x^2 \right]_0^2 \\ &= 48 \end{aligned}$$

2.2 Beberapa Contoh Soal

Solution

$$\begin{aligned} 2) \int_{-1}^1 \int_1^2 (x^2 + y^2) dx dy &= \int_{-1}^1 \left[\frac{x^3}{3} + xy^2 \right]_1^2 dy \\ &= \int_{-1}^1 \left[\left(\frac{2^3}{3} + 2y^2 \right) - \left(\frac{1}{3} + y^2 \right) \right] dy \\ &= \int_{-1}^1 \left[y^2 + \frac{7}{3} \right] dy \\ &= \left[\frac{y^3}{3} + \frac{7y}{3} \right]_{-1}^1 \\ &= \left(\frac{1}{3} + \frac{7}{3} \right) - \left(\frac{-1}{3} + \frac{-7}{3} \right) \\ &= \frac{16}{3} \end{aligned}$$

2.2 Beberapa Contoh Soal

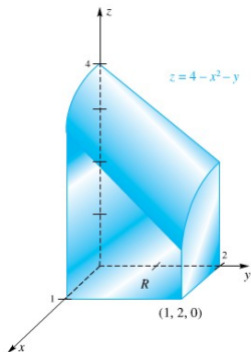
Solution

$$\begin{aligned} 3) \int_0^8 \int_0^4 \left[4 - \frac{x}{2} + \frac{y^2}{16} \right] dx dy &= \int_0^8 \left[4x - \frac{x^2}{4} + \frac{xy^2}{16} \right]_0^4 dy \\ &= \int_0^8 \left[4 \cdot 4 - \frac{4^2}{4} + \frac{4y^2}{16} \right] dy \\ &= \int_0^8 \left[12 + \frac{y^2}{4} \right] dy \\ &= \left[12y + \frac{y^3}{12} \right]_0^8 \\ &= \left[12 \cdot 8 + \frac{8^3}{12} \right] \\ &= 138 \frac{2}{3} \end{aligned}$$

2.3 Volume Benda Pejal

Example

Hitunglah volume benda pejal dibawah permukaan $z = 4 - x^2 - y$ diatas persegipanjang $R = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 2\}$.



2.3 Volume Benda Pejal

Solution

- *Volume estimasi :*

Misal tinggi benda pejal = 2.5, maka diperkirakan

$$V \approx (2)(2.5) = 5$$

- *Volume dengan pendekatan integral berulang:*

Jika perhitungan benar, maka volume yang diperoleh harusnya tidak jauh dari angka 5.

$$\begin{aligned} V &= \iint_R [4 - x^2 - y] dA \\ &= \int_0^1 \int_0^2 [4 - x^2 - y] dy dx \end{aligned}$$

2.3 Volume Benda Pejal

Solution

$$\begin{aligned}V &= \int_0^1 \left[4y - x^2y - \frac{y^2}{2} \right]_0^2 dx \\&= \int_0^1 [6 - 2x^2] dx \\&= \left[6x - \frac{2x^3}{3} \right]_0^1 \\&= 6 - \frac{2}{3} \\&= \frac{16}{3}\end{aligned}$$

2.3 Latihan 1

Problem

1. Hitunglah masing-masing integral berulang dari

$$1) \int_0^3 \int_0^2 (9 - x) \, dx dy$$

$$2) \int_1^2 \int_{-1}^1 (x^2 + y^2) \, dy dx$$

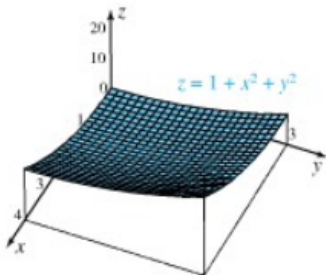
$$3) \int_0^4 \int_0^8 \left[\frac{64 - 8x + y^2}{16} \right] dy dx$$

2.3 Latihan 1

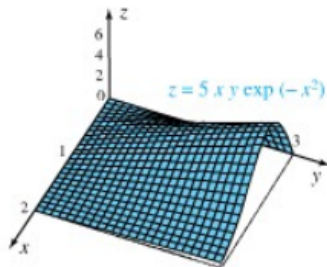
Problem

2. Hitunglah volume benda pejal dibawah permukaan:

23.



24.



" Terima Kasih, Semoga Bermanfaat "