IV. MATHEMATIC EDUCATION

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Students' Concept Image of Permutation and Combination viewed from Difference of Gender with High Ability in Basic Mathematics

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ABSTRACT

In order to understand the formal concept of combination and permutation and formal solution of its problem solving, which is presented in a lecture, it forms an institutionalized way of understanding mathematics. A student has to interpret the expression of definition as a personal interpretation of the definition. These interpretations are essential factors in the personal way of understanding the formal concept of combination and permutation. The formal concept definition is usually unambiguous, but the personal interpretations of the definition may vary between individuals, and they may also depend on the context. The latter is just not necessarily based on the formal definition. It can be thought that every time when a student applies a formal definition in a reasoning, the student in fact applies his/her personal interpretation of the definition. The personal interpretation of the concept definition is part of the student's concept image. This study conducted in a qualitative approach to some of the mathematics students, it shown that concept image of the mathematics student of the concept of combination and permutation, there exists a different to express the concept image for male and female student with high ability in basic mathematics.

Keywords: mental picture, concept image, relational concept image, instrumental concept image

1. Introduction

Mathematics is commonly associated with an abstraction, due to the theoretical concepts, sometimes being difficult to visualize and concretize. As a consequence, abstraction has been given a lot of interest in mathematics education to be a major goal of learning mathematics. The way the student adopts and interprets a mathematical concept also depends on influential factors that characterize students' thinking, like earlier achieved knowledge and experience as well as the situation where the concept occurs. This means that the student visualizes the concept and creates a symbol or a certain mental model. The concept image is one of the mental models that used of the students in learning mathematics of a formal concept.

In the theory developed by [1], the *concept image* is the total cognitive structure associated to a mathematical concept in an individual's mind. It includes all the mental pictures, properties, mental associations and processes related to a given concept, and is continually constructed and changing with all kinds of experience. Moreover, the concept image may (or not) be associated to a statement used to specify that concept, named the *concept definition*. A concept definition, in its turn, may (or not) be coherent with the formal mathematical definition. [1] have defined the *concept definition* to be a form of words used to specify the concept. The concept definition (formal definition) generates its own concept image, they call as a *concept definition image*, which is separated from a *formal concept definition*. It may also be a personal reconstruction by the student of a definition and learnt by an individual in a rote and related to a greater or lesser degree to the concept as a whole. The conflicts between students' concept image and the "formal definition" of that concepts are known as cognitive conflicts [1].

In [2] has also presented results of students' difficulties to use concepts in a constructive correct manner. This means that some students have an intuitive sometimes procedural conception of the concepts and need guidance to take the steps to formalise their knowledge. Even, in [3] pointed out that "the concept image can be considered as part of intuition" and intuitive ideas are an important and decisive part of a concept image. But, in order to understand the formal concept definition, such as in a lecture or in a text book, an individual usually makes an interpretation of the expression in the own definition. It is called as a *personal interpretation of the definition*. These interpretations are essential factors in the *personal way of understanding* mathematics.

Understanding a concept means that an individual is able to connect that certainconcept to his or her concept image in a significant way [4] which is different from just being able to perform a particular operation. [5] described two ways of understanding a concept, trough formal or natural learning. A formal learner uses definitions and symbols as a ground, whereas natural learners logically construct new knowledge from their concept images as a personal understanding. According to [6], in the matters for the understanding of mathematics, the personal understanding that is acquired by the students to be conceptions and possible misconceptions.

Combinatorics is an important field of research, it is the study of ways to list and arrange elements of discrete sets according to spesified rules, with vast applications in mathematics and other fields of science [7]. This study is based on the analysis of an interview of one male student and one female student of high ability in basic mathematic who had in the written test answered of the concept of permutation and combination that they understand it. And the aim of this study is to descript a profile of mathematics student's concept image of combination and permutation viewed from gender difference for student with high ability in basic mathematics.

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2. Theoretical approach

According to [1], the concept image may not be globally coherent and may have aspects which are quite different from the formal concept definition, in this way a *personal* concept definition (concept definition image) can differ from a *formal* concept definition. The concept definition image means an individual's personal way to define the concept in practice, whereas the formal concept definition is part of the formal axiomatic system of mathematics, which is accepted by the mathematical community at large. This system consists of axioms, definitions, rules of logic, and mathematical language, and it forms an *institutionalized way of understanding mathematics* [2]. Even when the concept is presented to or constructed by an individual, he could have his personal concept definition which can differ from the formal concept definition. The formal concept definition is usually unambiguous, but the personal interpretations of the definition may vary between individuals, and they may also depend on the context [8],[9],[10]. However, the personal interpretation of the definition does not mean the same as the personal concept definition. Because the personal concept definition is not necessarily based on the formal definition at all. It can be thought that when an individual applied a formal definition in a reasoning, he/she in fact applied his/her personal interpretation of the definition. Thus, the personal interpretation of the concept definition is part of the concept definition image, which, for one, is part of the whole concept image. A diagram describing the internal structure of the concept image in the relationship of institutionalized way of understanding and personal way of understanding is presented in Figure 1

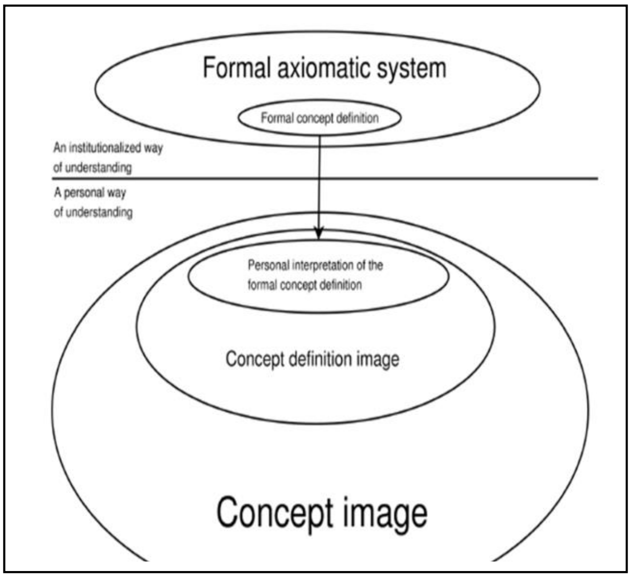

Figure 1: *The structure of concept image in the relationship between institutionalized and personal way of understanding [2]*

[1] use the term concept image "to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and "processes". The mental picture of a concept is the set of all pictures that have ever been associated with a concept in mind.The term "process" will be used in a general sense, as in the "process of addition", the "process of multiplication", the "process of solving an equation", to mean a cognitive or mathematical process [11].

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The *properties* are expressed in terms of formal definitions that are used as axioms to specify mathematical structures, prepositions, theorems, with the concept constructed by logical deduction.

The individuals build up their concept images in a way that may not always be coherent and consistent with the formal definition, and it depends on their previous experiences when they are met in new contexts. Therefore, the concept image differs between individuals and is reflecting personal reconstructions of a definition which generates individual perceptions. In consequence, the students can have a cognitive conflict or misconception in their concept images with the formal concept definition. Despite it, the concept image of the students have a certaintly levels. [12] indentified there are three levels of concept image of the students, namely *incipient concept image*, *instrumental concept image and relational concept image*. These levels are based on the basis of the objects, processes, translation between representations, properties and proceptual thinking that the students reveal their answers to the cognitive tasks that are placed to it.

Based on this model with concept definition and concept image, [13] explained that a misconceptions or a cognitive conflict can occur in learning situations as a consecuence of the relationship between concept image and concept definition that is reciprocal and mutual. However, teachers tend to believe that during this process, it is a one-way relationship, namely from the concept definition towards the concept image.

Clearly the variety of concept images can help learning by suggesting ways of giving students a more coherent concept image. Because, having a coherent concept image can help students achieve a more complete understanding of the concept, which is what will keep remain in their mind [14]. According to [2], some criteria for a high level of coherence of a concept image consists of a clear conception about the concept; all conceptions, cognitive representations and mental images concerning the concept are connected to each other; concept image does not include internal contradictions, like contradictory conceptions and a concept image does not include conceptions which are in contradiction with the formal axiomatic system of mathematics.

3. Methodology

This study is a qualitative approach to get student concept images of combination and permutation. The subjects of the research are third semester students from mathematics study program of faculty of mathematics and natural science, Hasanuddin University, Makassar, had taken the course of Basic Mathematic I and II and have concept images of permutation and combination in their mind, namely each one male (LT) and female (PT) student who has high ability in basic mathematic. These subjects are chosen with the criterions: The student shoud be able to reveal his or her mental pictures, processes and properties associated with concept of permutation and combination from his or her mind by external representation, able to

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explain the meaning of external representation that he or she was made. Collecting of the data from mathematic ability of the student conducted with giving a Mathematic Ability Tes (TKM). From the mathematics students of third semester who have done of semester final examination, then they will be given a basic mathematic test with total score 100, and it consist of limit of function, derivatif of multivariable function, integral, determinan and matrix. The reliability test of the instrument was used the correlation coefficient test of Alpha Cronbach to the scores of the test. Whereas, to determine the subject of the research, researcher choose one male student and one female student who have a score greater than 85.

While for collecting of the data from concept images of permutation dan combination conducted with giving a written test of Permutation and Combination (TKP) and interview by researcher. The written test and interview were designed in order to provide the students with an opportunity to express thoughts and conceptions of combination and permutation. Interview was based on the anwers of written test that is given by interviewee. The written test consist of 6 problems about the conception of combination and permutation, how to change a solution from the forms combination solution to permutation solution and vice versa, how to show the combination and permutation formula held and to extending its formula.

4. Results

From the score of basic mathematic test of 31 mathematics students of third semester and using the correlation coefficient test of Alpha Cronbach to the score results of the test, it was obtained the correlation value r=0.5715 and according to [15] the instrument is moderate. From these scores, researcher choose one male student and one female student who have the greatest score. While, for looking the concept image in subject's mind, it can be looked through the external representations of the subject that is made.

While the results of this study as descripting the profile of concept image of male and female subject who have high ability in basic mathematics to the concept of permutation and combination, namely:

(1) ***Concept image of the male subject* (LT)**, for *the concept of permutation k object of n object*:

The mental picture of subject LT about permutation k objects of n objects, *first* as a sequence of finite abstract objects, that is shown visually and symbolicly in the form a concequtive k boxes that contain abstract symbols of elements of a set n elements. However, subject LT has undergone a cogntive conflict to simbolizing any k objects as a permutation k objects of n objects. This mental picture is shown in the representtion in figure 1. The *second* mental picture, it is an ordered arrangement of finite element which in form of more one row, it is shown visually as a picture of arrangement k elements like a matrix. This mental picture is shown in the representtion in figure 2

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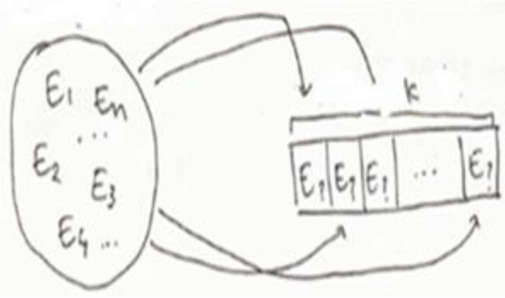
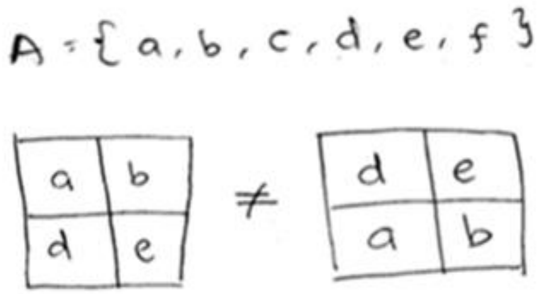


Picture 1: The first mental picture of permutation



Picture 2: The second mental picture of permutation

The process of the first mental picture is to drawing a connecting between a set of n elements and the consecutive of k boxes that contains any k elements from those set, it interprets as a sequence of k elements which is a required order. Subject LT used the symbol of permutation as sequence of elements, it was inspirated by arrangement of the letters in a sentence or arrangement of digits in a number. Subject LT used the properties that permutation is a finte ordered arrangement of the finite set and matrix is a ordered arrangement of finite element, therefore matrix is a permutation.

(2) ***Concept image of the female subject*** **(PT)**, for *the concept of permutation k object of n object*:

The mental picture of subject PT about permutation k objects of n objects, the *first* as a sequence of finite concrete objects, that is shown visually and symbolicly in the form a concequtive k boxes that contain the concrete symbols of a set n concrete objects. This mental picture is shown in the representtion in figure 3. However, The *second* mental picture, it is a arrangement of finite element which in form of more one row, it is shown visually as a picture of arrangement k elements like a matrix. This mental picture is shown in the representtion in figure 4

Picture 3: The first mental picture of permutation of subject PT

Picture 4: The second mental picture of permutation of subject PT

The process of the first mental picture is to drawing a connecting between a set of n concrete objects and the consecutive of k boxes that contains any k conrete objects from those set, it interprets as a sequence of k elements which is a required order. Subject PT used the symbol of permutation as sequence of elements likethe symbol of k tuples of elements, it was inspirated by arrangement of elements of the vector in the coordinate of Cartesian. Subject PT used the properties that permutation is a finite ordered arrangement of the finite set and the elements should not be different

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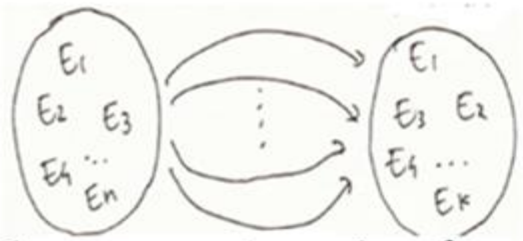
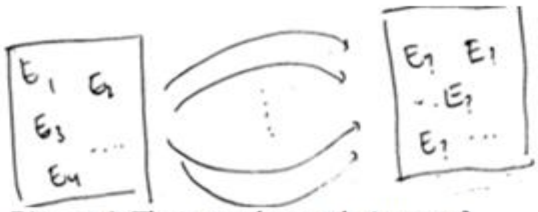
and the arrangement of the elements in permutation can in form of any arrangement of elements, not only in one row of elements, but also in arbitrary form of arrangement of elements that the order is required.

(3) ***Concept image of the male subject* (LT)**, for *the concept of combination k object of n object*:

The mental picture of subject LT about combination k objects of n objects, the *first* as a subset of k elements of a set n elements, that is shown visually and symbolicly in the closed curve that contain abstract symbols of elements of a set n elements. However, subject LT has undergone a cogntive conflict to simbolizing any k objects in the subset as a combination k objects of n objects (picture 5). The *second* mental picture, it is a result of selecting, mixing or taking of the finite elements to become an unordered arrangement of finite elements (picture 6).



Picture 5: The first mental picture of combination of subject LT



Picture 6: The second mental picture of combination of subject LT

The process of the first mental picture is to drawing a connecting between a set of n elements and a subset as a collecting of any k elements from those set, it interprets as a combination k elements which no requiring the order of elements. Subject LT used the symbol of combination as a collecting of the finite elements, it was inspirated by the symbol of the subset. Subject LT used the properties that combination is a finite unordered arrangement of the finite set and a set is any unordered collection of distinct objects. However, subject LT has undergone misconception about elements in the combination, because in the combination is recognized the repeated combination. This occur, because subject LT has a mental picture of the combination as a subset which the elements should be different. This subject tends to use an abstract symbol to symbolize all elements in its combination. Subject's concept definition of combination k object of n object is consistent with formally definition, but not consistent with his concept image.

(4) ***Concept image of the female subject* (PT)**, for *the concept of combination k object of n object*:

The mental picture of subject PT about combination k objects of n objects, the *first* as a subset of k elements of a set n elements, that is shown visually and symbolicly in the closed curve that contain concrete objects of a set n concrete

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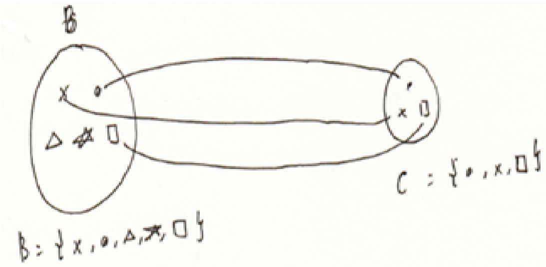
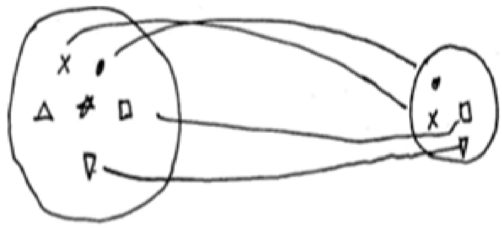
objects. However, subject PT has undergone a cogntive conflict to simbolizing any k objects in the subset as a combination k objects of n objects. This mental picture is shown in the representtion in figure 7. The *second* mental picture, it is a result of selecting of the finite objects to become an unordered arrangement of finite objects. This mental picture is shown in the representtion in figure 8.



Picture 7: the first mental picture of combination of subject PT



Picture 8: The second mental picture of combination of subject PT

The process of the first mental picture is to drawing a connecting between a set of n concrete objects and a subset as a collecting of any k objects from those set, it interprets as a combination k elements n elements. Subject PT used the symbol of combination as a collecting of the finite elements, it was inspirated by the symbol of the subset. Subject PT used the properties that combination is a finite unordered arrangement of the finite set and that the elements in subset should be different. However, subject PT has undergone misconception about elements in the combination, because in the combination is recognized the repeated combination. This occur, because subject PT has a mental picture of the combination as a subset which the elements should be different. While, subject's concept definition of combination k object of n object is consistent with formally definition, but not consistent with his concept image.

(5) ***Concept image of the male and female subject (LT and PT)*** , for the concept to solve the problem with combinationly solution to solve the problem with permutatingly solution

Problem: *12 persons will be placed in 3 different cities, each consists of 3 persons, 4 persons and 5 persons. By using combination way and then permutation way, calculate how many ways to place these persons.*

The mental picture of subject LT about changing a solution of the problem with the combinationly to the permutationly. It is to change the solution of the problem by making partition of the set n elements. This mental picture is shown in representtion in figure 9 and 11, to become the ordered arrengement of n elemen which it consists of element repetitions, it is shown in representasion of mental picture in figure 10 and 12

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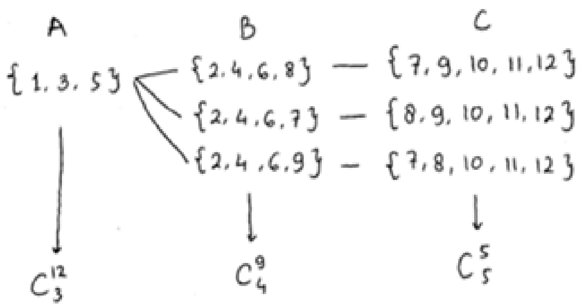

Figure 9: the mental picture of problem solving with combinationly solution of subject LT

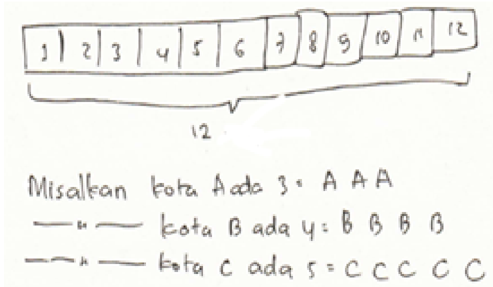

Figure 10: the mental picture of problem solving with permutationly solution of subject LT

Subject LT and PT usually used the formula combination and knew that the construction of the solution should use partition of the set, and it is a selection elements which should be placed in three different places, so that the solution as a multiplication of combination. This mental picture is shown in the representtion in figure 9 and figure 1. However, subject LT and PTlooks that the contruction of this solution can be looked as choosing 3 positions number for persons in first place, 4 position number for person in second place and 5 positions number for persons in third place from n concecutive positions so that the solution as a permutation with repetition, The mental pictures are shown in the representtion in figure 10 and figure12. Subject LT and PT used the properties that the number of way of making subset k elements from a set n elements is equaly the number of combination k objects from n objects. The number of way for choosing some objects from n object is equaly the number of way of arranging that some objects in an ordered n positions.

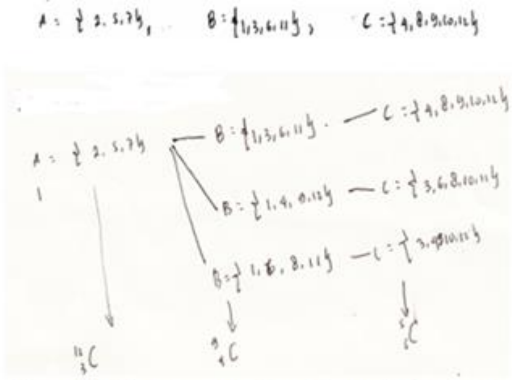

Figure 11: The representation of mental picture of the combinationly solution of subject PT

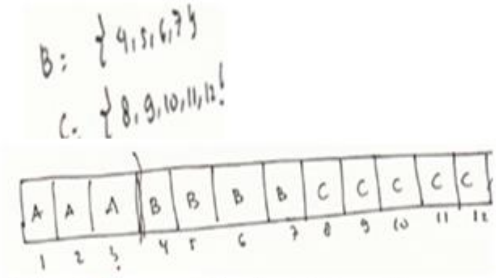

Figure 12: The representation of mental picture of the permutationly solution of subject PT

(6) ***Concept image of the male and female subject (LT and PT)***, for *the concept solving the problem with permutation way solution into solving the problem with combination way solution*

Problem: How many words can be made that consist of two ‘A’, three ‘B’ and five ‘C’

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The mental picture of subject LT and PT about changing a solution of the problem with the permutation way solution to a combination way solution is to changing the solusion of the problem by making the ordered arrengement of n elemen which it consists of element repetitions. This mental picture is shown in representtion in figure 13 and 15. While the combination way solution is a partition of the set n positions. It is shown in the representasion of mental picture in figure 14 and 16

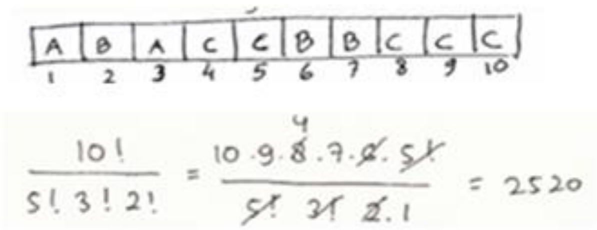

Figure 13: Representation of mental picture of permutation way solution of subject LT

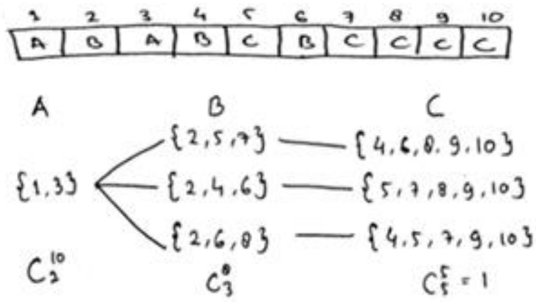

Figure 14: : Representation of mental picture of combination way solution of subject LT

Subject LT and PT familiarly used the formula of permutation with repetition, this mental picture is shown in the representtion in figure 13 and figure 15. They knew that the permutation way solution can be looked as a solution of the selection of different positions for the same elements from a set of n positions. Theefore, this way is a combination way solution. This mental picture is shown in the representtion in figure 14 and figure 16. Subject LT and PT used the properties that the number of way of making subset k elements from a set n elements is equaly the number of combination k objects from n objects. The number of way for choosing some objects from n object is equaly the number of way of arranging that some objects in an ordered n positions.

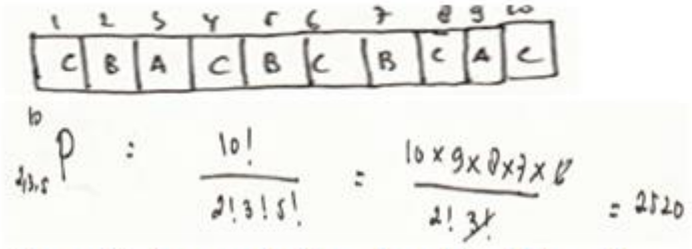

Figure 15: Representation of mental picture of permutation way solution of subject PT

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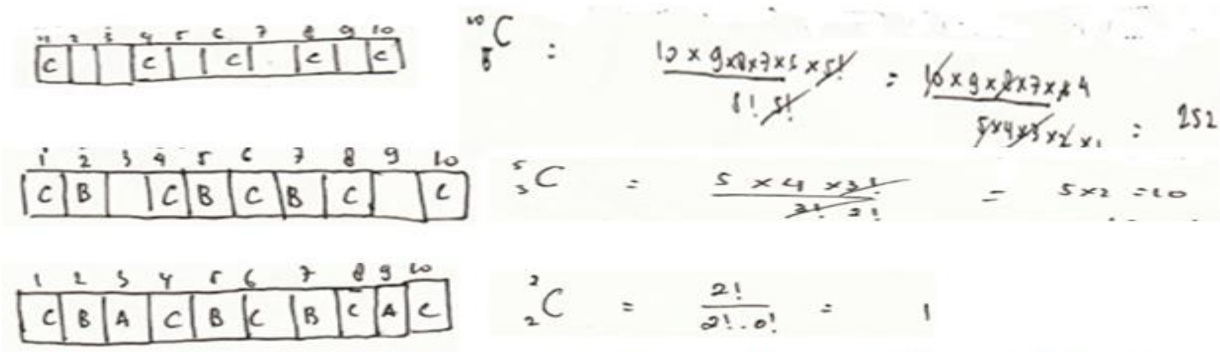

Figure 16: representation of mental picture of combination way solution of subject PT

(7) ***Concept image of the male and female subject (LT and PT)***, for *proving the combination equality formula*

Problem: *Let "x" be one element of n different elements, such that for any combination k elements of n elements will contain or not contain "x", prove the formula* $C_k^n = C_k^{n-1} + C_{k-1}^{n-1}$ *and how to expand the formula!*

The *first* mental picture of subject LT to prove combination equality formula is to formulate the number of way for making subset k elements from a set n elements with one certain specific element, which all alements are symbolized by using abstract symbols, but he did not use a symbol that shows a symbol for any k elements in the subset k elements. While, subject PT for symbolizing any elements of subset k elements, she used symbol 'dot' and '?' except for specific element (figure 19). Thereby, subject LT and PT have undergone a cognitive conflict to symbolize any k elements in the subset k elements (figure 17 and19). Subject knew that for making any subset k elements from a set n elements with one certain specific element, the subset contains or not contains the specific element. Therefore, subject LT and PT knew that is a independently event and they used the combination formula to count the number of way for making each kind of subset with using principle of inclusion in the set. The mental picture is shown as a visually representation in figure 17 and figure 19. While, the *second* mental picture for proving the combination equality formula, subject LT and PT attemp to prove that the left and right side of formula is equal. This mental picture is shown with using algebra manipulation to prove this formula (figure 18) , such that two concept image as above is coherent. Thereby, for proving this formula, subject LT and PT have related to the other concept, namely, principle of independently event, set theory and principle of inclusion-exclusion.

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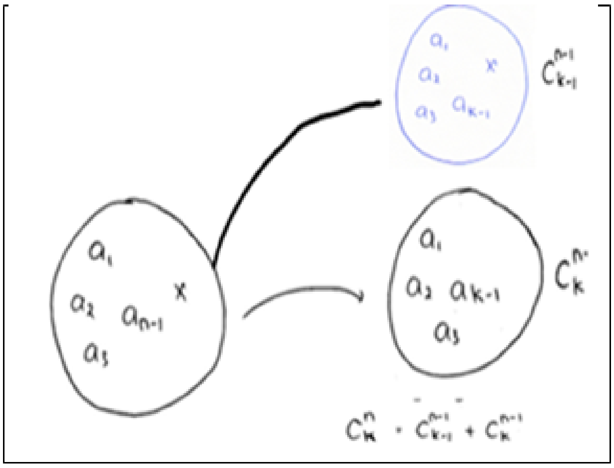

Figure 17: The first representation of mental picture for profing the combination eauality formula of subjek LT

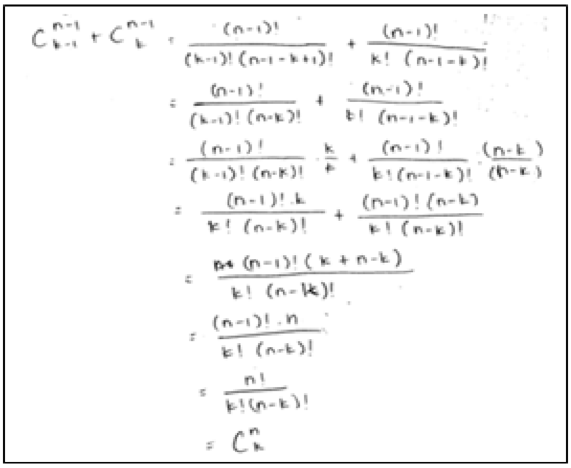

Figure 18: The second representation of mental picture for profing the combination eauality formula of subjek LT

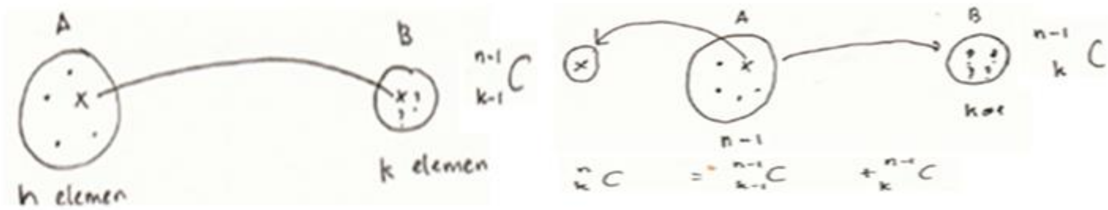

Figure 19: Representation of the first mental picture for proving the combination equality formula of subject PT

While, the mental picture of subject LT and PT for expanding the formula is to formulating the number of way of all subsets k elements from a set n elements with containing more of one certain specific element. This mental picture is shown in the visually representation in figure 20 and 22. However, subject LT and PT has also undergone cognitive conflict for symbolizing any k elements besides the specific elements.

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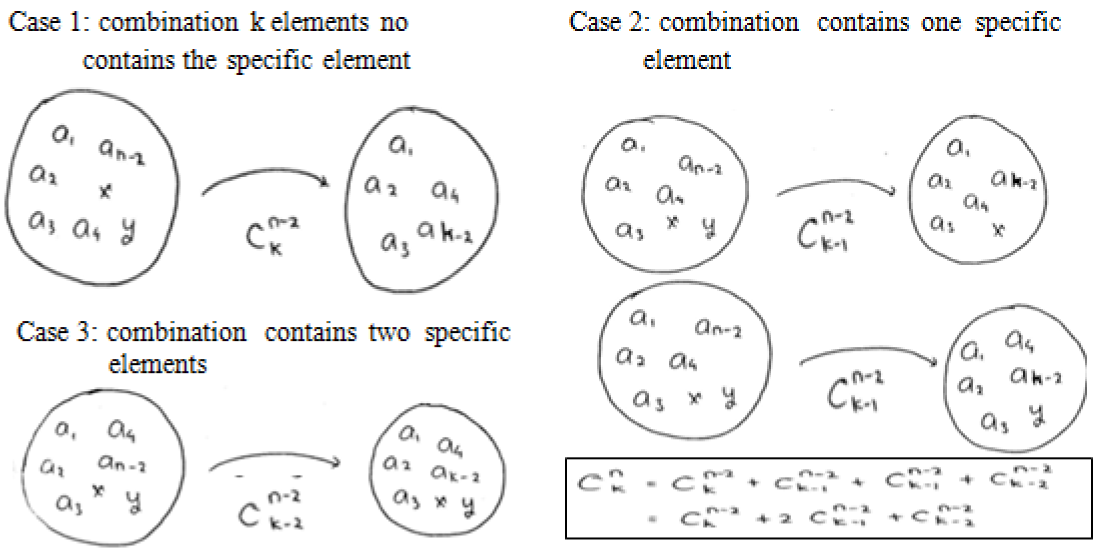

Figure 20: Representation of mental picture for expanding of the combination equality formula of subject LT

Subject LT and PT knew that all coefficients in the expanded formula forming a Pascal number, therefore for more expanding the formula, subject used the Pascal number for all coefficients and combination formula in the expanded formula (figure 21 and 23). Thus, subject LT and PT has related for expanding of the formula to the principle of inclusion-exclusion in the set and principle of Pascal number.

$$C^n_k = C^{n-3}_{k-3} + 3C^{n-3}_{k-2} + 3C^{n-3}_{k-1} + C^{n-3}_{k}$$
$$C^n_k = C^{n-4}_{k-4} + 4C^{n-4}_{k-3} + 6C^{n-4}_{k-2} + 4C^{n-4}_{k-1} + C^{n-4}_{k}$$

Figure 21: Expanding of the combination equality formula of subject LT

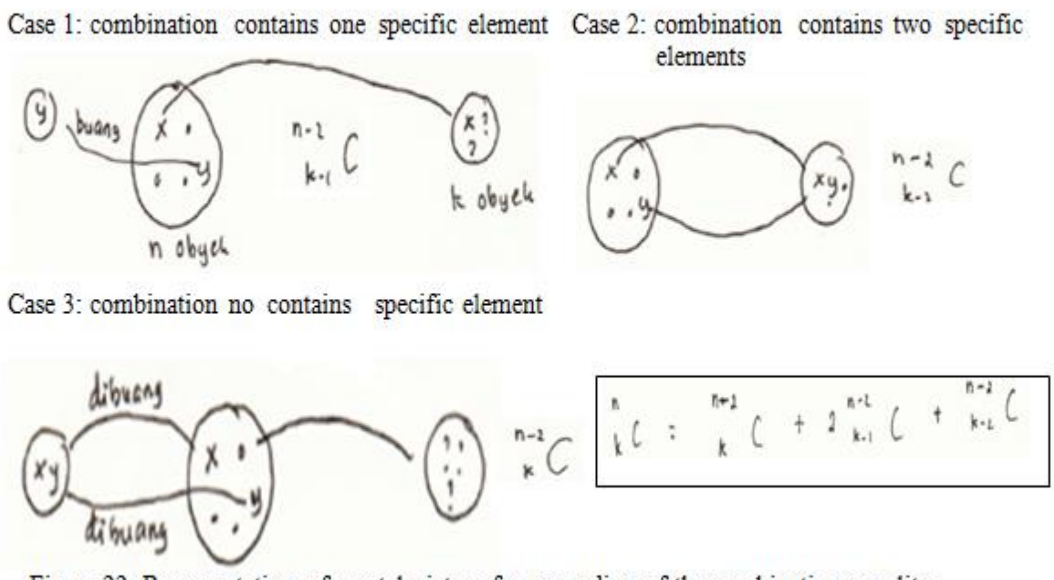

Figure 22: Representation of mental picture for expanding of the combination equality formula of subject PT

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Figure 23: Expanding of the combination equality formula of subject PT

(8) *Concept image of the male and female subject (LT and PT)*, for *proving the permutation equality formula*

Problem: *Let "x" be one element of n different elements, such that for any permutation k elements of n elements will contain or not contain "x", prove the formula* $\boldsymbol{P}_k^n = \boldsymbol{k}\,\boldsymbol{P}_{k-1}^{n-1} + \boldsymbol{P}_k^{n-1}$ *and how to expand the formula!*

The first mental picture of subject LT to prove permutation equality formula is to formulate the number of way of k elements the ordered arrangement from a set n elements with one certain specific element, which all alements besides the specific element are symbolized by using 'blank' and 'dot' symbols, he did not use a symbol that shows a symbol for any k elements in the ordered arrangement of k elements (figure 24). While, subject PT for symbolizing any elements of subset k elements, also used 'blank'symbol (figure 25). Thereby, subject LT and PT have undergone a cognitive conflict to symbolize any k elements in the subset k elements . Subject knew that for making any ordered arrangement of k elements from a set n elements with one certain specific element, the ordered arrangement contains or not contains the specific element. Therefore, subject LT and PT knew that the set of all permutations that contains the specific element and the set of all permutations that not contains the specific element is disjoint and they used the permutation formula to count the number of way for making each kind of permutations with using principle of inclusion in the set. The mental picture is shown in the visually representation and depicted in figure 24 and figure 25. While, the second mental picture for proving the permutation equality formula, subject LT and PT attemp to show that the left and right side of formula is equal. This mental picture is shown with using algebra manipulation to prove this formula, such that two concept images as above coherent. Thereby, for proving this formula, subject LT and PT have related to the other concepts, namely, principle of independently event, set theory and principle of inclusion-exclusion.

Case 1: the ordered arrangement of k elements that contains a certain specific element

Case 2: the ordered arrangement of k elements that no contains a certain specific element

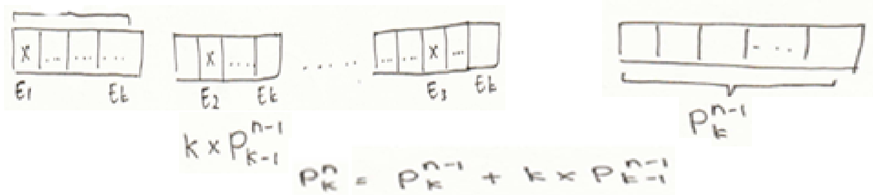

Figure 24: The frist representation of mental picture for proving of the permutation equality formula of subject LT

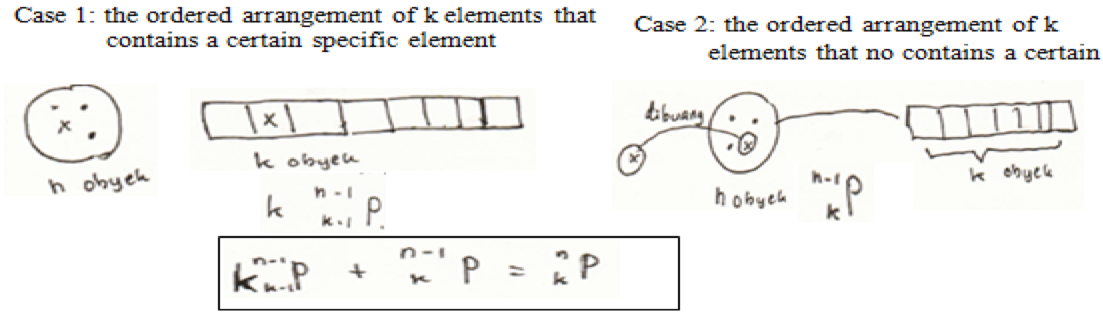

Figure 25: The first representation of mental picture for proving of the permutation equality formula of subject PT

Whereas, the mental picture of subject LT and PT for expanding the formula is to formulating the number of way of all ordered arrangements of k elements from a set n elements with containing more of one certain specific element. This mental picture is shown in the visually representation in figure 26 and 27. However, the figure of subject LT is more variety than the figure of subject PT. Subject PTand PT has also undergone cognitive conflict for symbolizing any k elements besides the specific elements.

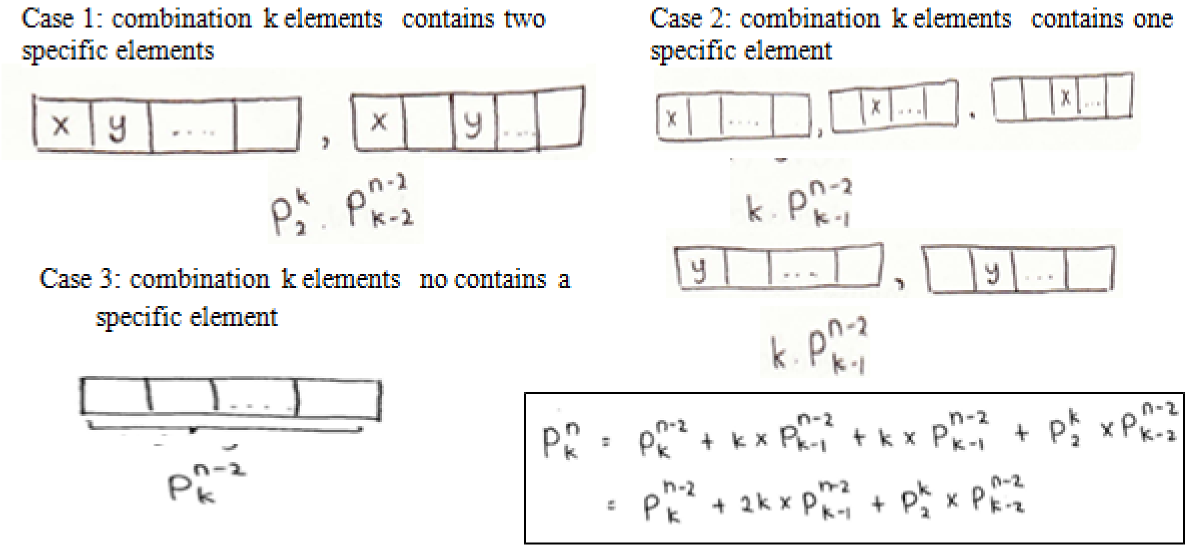

Figure 26: Representation of mental picture for expanding of the permutation equality formula of subject LT

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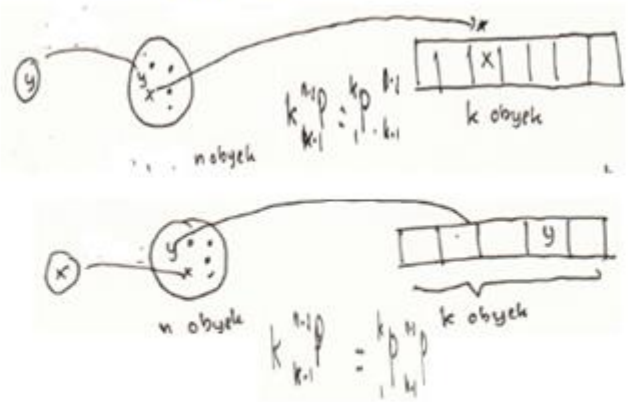
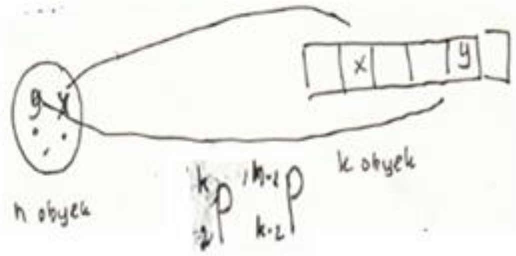
Case 1: combination k elements contains one specific element



Case 2: combination k elements contains two specific elements



Case 3: combination k elements not contains a specific element

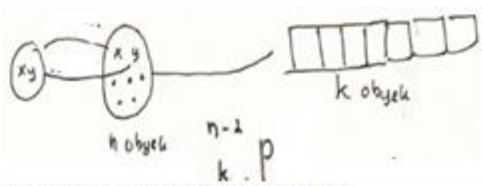


Figure 27: Representation of mental picture for expanding of the permutation equality formula of subject PT

Subject LT and PT knew that all coefficients in the expanded formula forming a Pascal number, therefore for more expanding the formula, subject used the Pascal number for all coefficients and permutation formula in the expanded formula (figure 28 and 29). Thus, subject LT and PT has related for expanding of the formula to the principle of inclusion-exclusion in the set.

$$P^n_K = P^K_3 \, P^{n-3}_{K-3} + 3\, P^K_2 \, P^{n-3}_{K-2} + 3\, P^K_1 \, P^{n-3}_{K-1} + P^K_0 \, P^{n-3}_K$$

Figure 28: Representation of mental picture for more expanding of the permutation equality formula of subject LT

Figure 29: Representation of mental picture for more expanding of the permutation equality formula of subject PT

5. Discussion

From the results, the male subject described the concept image of the permutation and combination tends to use abstract symbol, while the female subject tend to use concrete symbol, but both of the subject have same the concept image of the permutation that the permutation is as sequence of the finite objects selected from finite objects and the subjects are only inspirated by the arrangement of letters in a sentence or digits in an integer. This is a fact, that their concept image is influenced with their previous life experience about a consecutive things that required the order of the things. Therefore, according to [12], the concept image of the permutation for both subject are categorized *instrumental concept image*. While, for the combination is as finite subset of the objects from the given finite objects. Therefore, from this concept image of the combination, both subject have assumed that in combination all objects are different, it cause a misconception of the subject about the combination. Both subject has also undergone a cognitive conflict to symbolize any k elements that is taken from a set n elements as arbitrary permutation k objects or combination k objects of n objects. However, or the concept image of combination, both subject attempted to involve with the properties of finite set or finite subset. Thus, according to [12], this concept image can be categorized as *relational concept image*.

For both subject, the concept image of the problem solving for changing the combination solution become the permutation solution is to change the solution with making partition of the set in disjoint finite subsets then become the ordered objects arrangement with finite repetition of these objects. Thus, both subject has used the concept of the partiton of the set and concept of the ordered objects arrangement. Therefore, the concept image of the problem solving for changing the combination solution become the permutation solution is categorized *relational concept image*.

While, the concept image of the problem solving for changing the permutation solution become the combination solution is to change the solution with making the ordered objects arrangement with repetition, then become the number of way for selecting and collecting the position numbers for the same objects in a subset, such that will be built a partition of the set of partition number. Thus, both subject has involved with the concept of the ordered objects arrangement and concept of the partition of the set. Therefore, this concept image is categorized *relational concept image*

The concept image of both subject for proving the combination (permutation) equality formula is shown through two ways, the first it used the collection of subsets (finite tuples) with contains a specific element and the collection of subsets with not contains a specific element, the second it used algebra manipulation, that right side can be equalized to the left side formula. Both concept image is coherent and they have undergone a cognitive conflict to symbolize any k elements in the subset from set n elements as any combination (permutation) k object of n objects. This is caused the subjects have no experience to express any subset of a subset in the symbol formally. However, for expressing their concept image, the male subject more variety and obvious than female subject in using the representation visually and symbolicly. The concept image of both subject for expanding the combination (permutation) equality formula also showed that they developed the proccess for proving the formula with using collection of kinds of the subsets, but for further expanding the formula, both subject connected to principle of inclusion-exclusion and how to build the Pascal number. Thus, the concept image of both subject is categorized *relational concept image*.

6. Conclusion

On basicly, the concept images of male and female student to the permutation or the combination who has high ability in the basic mathematics are not different, they are influenced with their previous life experience, even also for the category of their concept image. Even, they have more than one of the concept image, but these concept images are coherent. They only have a difference in way to express their concept image into external representation visually, the male subject tends to express in abstract symbols, while the female subject tends to express in concrete symbols. While, the concept image of male and female student who high ability in basic mathematics to the problem solving of combination and permutation is not different. They have a previous experience to solve the problem solving of combination with using the concept of selecting and collecting objects from a finite set, such that forms a partition of the set. And also, to change the combination solution to become the permutation solution, they have no different concept image.

While, the concept image for proving the combination and permutation equality formula of both subject is not different. They have more They only have a difference in way to express their concept image into external representation visually and symbolicly, the male subject tends to express more variety visually than the female subject. Both subject has more than one concept image and coherent, but they have also a cognitive conflict to symbolize any finite elements of the set as any combination or permutation finite objects of the collection of finite objects. Thus, the concept image of the male and female students who have high ability in basic mathematics, then they have more than one concept image and coherent.

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The Reduction of Student Misperception in Set Topic through Cognitive Conflict

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ABSTRACT

The purpose of this study is to reduce the students misperception in set topic through cognitive conflict. The subject of this study are students who fail in identifying the elements of set or in identifying the subsets of set. The method of this research is a qualitative research. This research includes four steps, these are consisting of giving pre test, identifying thinking process of subject of this research, giving cognitive conflict, and giving post test. This result of this research is the information that learning by cognitive conflict can reduce misperception of student in identifying the elements of set or in identifying the subsets of set.
Key Words: cognitive conflict, misperception, to reduce

1. Introduction

Mathematics is a deductive-axiomatic knowledge and constructed by some systems. Mathematics system includes some sets and some operations. Therefore, the set theory is one of fundamental topics of mathematics knowledge. Student of mathematics department must be master in set topic, in particular, subset and the membership of set. If not, they will get some difficulties to learn mathematics in the future. For example, if a student doesn't know the element of set $\{2z \mid z \in Z\}$, then he or she will get difficulties to determine whether $\{2z \mid z \in Z\}$ under addition is a group or not. Therefore, we must be immediately try to reduce student misperception. One of the effort can be applied is providing student cognitive conflict. It is choosen because when student get cognitive conflict, then they will get dissatisfaction (Kang dkk,2005). Naturally, someone will try to remove their dissatisfaction. The experience to reach the goal by removing dissatisfaction is more memorable than if it is not.

Based on the pervious explanation, there are two problems of this research. These are:

1. What is the description of cognitive conflict learning in order to reduce student misperception?
2. And the second is what is the effect of student cognitive conflict?

According to the problems, there are also two objectives of this research. The first objective is to describe cognitive conflict learning in order to reduce student misperception. The scond objective is to get information of the effect of student cognitive conflict.

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2. Methodology

The method of this research is a qualitative research. This research includes four steps, these are consisting of giving pre test, identifying thinking process of students as subject of this research, giving cognitive conflict, and giving post test. Pre test is given to get information of student initial perception of the member and subset of some sets. Based on pre test result, the subject of this research was choosen. Subject of this research is four students that failed in pre test and has good capability to communicate their idea. The second and the third steps, i.e. identifying thinking process subject of this research, and giving cognitive conflict are to be done by interviewing subject of this research . The information of misperception reduction is obtained by giving post-test. The problems of post test is equivalent to the problems of pre-test result.

3. Result and Discussion

3.1 Description of Pre-Test Result

The description of pre-test result is explained on Table 1

Tabel 1. Pre-Test Result

Problem	Description
1. Let $A = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in 3\mathbb{Z}\right\}$, $B = \left\{\begin{pmatrix} k & 2k \\ l & -l \end{pmatrix} \middle| k,l \in 3\mathbb{Z}\right\}$, and $C = \left\{\begin{pmatrix} 3p & -3q \\ -3r & 3s \end{pmatrix} \middle| p,q,r,s \in \mathbb{Z}\right\}$. Write two elements of each set $A, B,$ and C	• There were no student that answer by giving example of the element of set. The answer of each student was 2 element-subset of set A, B, and C. For example: Two elements of set A is $\left\{\begin{pmatrix} 3 & -3 \\ 0 & 6 \end{pmatrix}, \begin{pmatrix} -3 & -3 \\ 0 & -6 \end{pmatrix}\right\}$
2. Let $A = (-\infty, -6]$, $B = \{5z \mid -1 < z < 5, z \in \mathbb{Z}\}$, $C = [-1,5]$, $D = (-1,5)$, $E = \{-1,0,2,3,4,5\}$, $F = (-\infty, -5)$, $G = \{[2,7], -1, -2\}$. Determine the truth of each following statement! a. $C = \{D, -1, 5\}$ b. $-5 \in A$ c. $F \subseteq A$ d. $E \subset D$ e. $-5\frac{1}{2} \in F$ f. $25 \in B$ g. $\emptyset \subseteq \mathcal{P}(C)$ h. $\{0\} \subseteq$ B i. $\{\emptyset\} \subseteq \mathcal{P}(E)$ j. $2 \in G$	• Problem no. 2a,2b,2e,2f, and 2j measure student knowledge of membership of set • Problem no.2c,2d,2g, 2h and 2i measure student knowledge of subset of set • In problem 2a and 2j, all student wrote in wrong answer. (their answer are True) • In problem 2f all student answer was correct • There were 40% student got in wrong answer in problem 2b and 2e. It because of their number sense. It was not because of their knowledge of membership of set. They considered -5 less than -6 and $-5\frac{1}{2}$ more than -5 • There were also 40% student got in wrong answer in problem 2c. It because of their number sense. It was not because of their knowledge of subset. • There were 100% student got correct answer in problem 2e and 2h • There were 100% student got incorrect answer in problem 2g and 2i.

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3.2 Description of Interview Result

Based on description of pre-test result, this study chosed four student as respondence. The chosen topics are problem 2a, 2j, 2g, and 2i. By doing interview, this study got depth information of their thinking process. This study classified interview result into intial perception result and cognitive conflict result

Initial perception result of problem 2a and 2j

All student had initial perception of {D,-1,5} as union of interval (-1,5) and {-1,5}. Because union of interval (-1,5) and {-1,5} is interval [-1,5] that similar with set C, then C={D,-1,5} is correct statement. Following conversation represents brief of interview example.

L (Lecturer)	: You said that C={D,-1,5} is correct statement, why?
S (Student)	: Because 0 in D and also in C.
L	: only 0?
S	: There are many numbers that has same property, such as -2,-3. All numbers in D has that property.
L	: Then?
S	: {D,-1,5}=[-1,5]=C
L	: I still confuse of your statement: {D,-1,5}=[-1,5] . Please give me more explanation.
S	: {D,-1,5} is union of D and {-1,5}, and D=(-1,5), and union of (-1,5) and {-1,5} is [-1,5]

Cognitive conflict result

After knowing their initial perception, writer want to change it by giving cognitive conflict. Following conversation and following photograph in Figure 1 represent brief of interview example.

L: Ok, it's clear for me. By the way, please list all element of {D,-1,5}
S: I can't
L: Why?
S: Because I cant list the element of D
L: Look at set {D,-1,5}. Which method that apply to writ that set?
S: Roster Method
L: Good. So, can you list all element of {D,-1,5} know?
S: O, ya!! There are: D,-1,and 5
L: Is 0 in set C of the satement 2a?
S: No
L: In fact, is 0 in C? .... (giving student cognitive conflict)
S: Yes... O, I know .... (student got cognitive conflict)
L: What do you know?
S: The correct answer is C≠{D,-1,5} .... (student removing misperception)
L: How about no. 2j?
S: It's same, 2∉G .... (student removing misperception)

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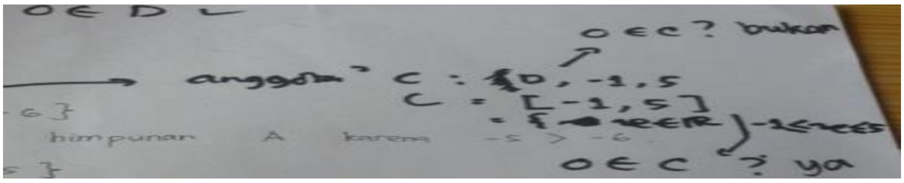

Figure 1. Student cognitive conflict of membership of set

<u>Initial perception result of problem 2g and 2i</u>

Following conversation represents brief of interview example.

L: Your answer of problem 2g is "False", why

S: Because ∅ subset of set C, so ∅ is an element of P(C) not a subset of P(C)

L: Remember, there is a theorem that state the relation of empty set and any set. Describe that theorem, please.

S: ....(Remembering)

L: Forgot?

S: Maybe... Empty set is subset of any set

L: Is P(C) a set?

S: Yes. Hmmh, so the correct answer "True", isn't it?

L: Yes, good. How about no. 2i?

S: I still do not understand of the meaning of {∅}

L: Look at set {2}. What is the element of {2}?

S: 2

L: So, What is the element of {∅}?

S: ∅. But there is no element of ∅. Is it possible ∅ become an element of set?

<u>Cognitive conflict result</u>

After knowing their initial perception, writer want to change it by giving cognitive conflict. Following conversation and following photograph in Figure 2 represent brief of interview example.

L: Let A={2}, what is P(A)? .... (giving cognitive conflict)

S: P(A)={2,∅} .... (student got cognitive conflict)

L: So?

S: yes, it is possible .... (removing misperception)

L: Can you find the answer of 2i?

S: {∅}⊆ P(E) is correct

L: why?

S: because ∅∈ P(E)

L: good, thank you

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3.3 Post Test Result

In order to make more confidence of misperception reducion, post-test has given to the respondence. The result gives us information that all respondence answer correctly.

3.4 Discuss

Student construct some consept if their scheme is suitable with the coming information. This condition is known as equilibrium. If the information is not suitable with their scheme, then they will be processed to get equilibrium of assimilation and accomodation (Cahyowati, 2009). Assimilation of Piaget is a process to integrate some concept into the existing scheme. Assimilation will be occured when student accept the new object or occurance (Bhattacharya and Han,2008). During processing of information, the assimilation is not always directly occured. Sometimes, student must modify their scheme before assimilating some informations. The scheme modification, in Piaget framework, is known as accomodation. Bayer (2006), express the accomodation as follows: Accommodation refers to the process of changing internal mental structures to provide consistency with external reality. It occurs when existing schemas or operations must be modified or new schemas are created to account for a new experience.

Another factor when constructing a concept is perception. Gregory (Leod, 2007) stated, perception involves making inferences about what we see and trying to make a best guess. Prior knowledge and past experience are crucial in perception. When we look at something, we develop a perceptual hypothesis, which is based on prior knowledge. The hypotheses we develop are nearly always correct. However, on rare occasions, perceptual hypotheses can be disconfirmed by the data we perceive.

If the information preceive different with student perception, then misperception will be occured. By giving cognitive conflict, student can realize their misperception. In mathematics learning, cognitive conflict can be occured when student realize their mathematics object perception is not suitable with the nature of mathematics object (Tall,1977). Cognitive conflict concept leads to Piaget Equilibrium Theory (Kang, 2005). Student who has cognitive confict indicate there is contradiction of the existing scheme and the information. Therefore, student who have cognitive conflict are in disequilibrium position. They feel uncomfort and dissatisfied, so they want to remove what they feel. Removing what they feel is similar with removing misperception. Physically, students gesture as a response of some contradiction can indicate the conflict cognitive.

The previous idea supports the result of this research. According to initial perception result of this research, it was known that student in equilibrium condition through assimilation process. In their scheme, contained idea that all elements of set $[-1,5]$ is also in set $\{(-1,5),-1,5\}$, and conversely. They also contained opinion that $2 \in \{[2,7],-1,-2\}$ because $2 \in [2,7]$ and $[2,7]$ is in $\{[2,7],-1,-2\}$.

But, when lecturer faced them to the fact that the method was used to write the set is roster method they saw a contradiction, and they modify their scheme that we call accomodation and got equilibrium again.

Eventhough at first they were in equilibrium condition, but they got misperception. Their initial perception contradict to the conceptual fact. Their initial perception were all elements of set $[-1,5]$ is also in set $\{(-1,5),-1,5\}$ but the fact is $0 \in [-1,5]$ and $0 \notin \{(-1,5),-1,5\}$; $2 \in \{[2,7],-1,-2\}$ but the fact is $2 \notin \{[2,7],-1,-2\}$. When they realize that the method was used to write the set is roster method, they faced to a contradiction, and they got cognitive conflict. They felt dissatisfaction and removed this dissatisfaction by change their initial perception. Therefore, their misperception was removed.

According to initial perception of identifying the subsets of set, the student perceive that $\emptyset \not\subseteq \mathcal{P}(C)$, because $\emptyset$ is an element of $\mathcal{P}(C)$. When lecturer confronted with the theorem "$\emptyset \subseteq A$, for any set A", they knew that their initial answer was incorrect. They looked surprised and happy, because they found new information, that was a set can be an element and subset of some set.

4. Conclusion

Based on the description of the results, this research conclude that learning by cognitive conflict can reduce misperception of student in identifying the elements of set or in identifying the subsets of set.

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THE ROLE OF COOPERATIVE LEARNING TO IMPROVE THE STUDENTS' MATHEMATICS COMMUNICATION AND PROBLEM SOLVING ABILITY IN THE SUBJECT OF PROBABILITY THEORY

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ABSTRACT

The importance of learning mathematics can not be separated from its role in all aspects of life. Communicating ideas by using mathematics language is even more practical, systematic, and efficient. In order to overcome the difficulties of students who have insufficient understanding of mathematics material, good communications should be built in a learning process. Communication in general can be interpreted as a way to convey a message from the messenger to the receiver to inform opinions or behaviors either directly (oral) or indirectly through the media. In the communication process, we need to think on how to make the message can be understood by others. In order to develop the ability to communicate, people can communicate with a variety of languages including mathematical language. One of the subjects in the Statistics study program which requires mathematics communication ability is the Theory of Probability, which is categorized as a general skill subject that must be taken by all students of mathematics. Improving the students' mathematics communication ability should be hand in hand with the learning process. We can optimize the ability by implementing a learning model which gives chance for the students to discuss and to interact each other so that their mathematics communication ability improved, that is by using cooperative learning type Team Assisted Individualization (TAI).

Keywords: Mathematics communication ability, Cooperative learning type TAI.

.

1. Introduction

1.1 Background

Mathematics learning at school does not only aim to make students understand the material. There are various objectives such as improving the mathematics reasoning ability, the mathematics communication ability, the connection of mathematics, the mathematics representation, and the mathematics problem solving, as well as some particular behaviors that should be internalized by students after they studied mathematics [2]. The mathematics communication ability is very important for the students so that they can solve mathematics problems by using good reasoning, illustrate the mathematics ideas into a mathematics model, and

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then connect the process into various mathematical concepts, into everyday life context, as well as into the other disciplines [5]. In studying the mathematics communication ability, the learning model which is predicted to be able to facilitate the study is the cooperative learning model type Team Assisted Individualization (TAI) [8]. This model is a group learning which puts the students in small groups consisting of two or more heterogeneous students to help each other in learning the material [9]. This model also emphasize on the positive interdependence among students, their individual responsibility, face to face meeting, intensive communication, and group evaluation process so that the classroom management becomes more effective [3].

The cooperative learning type TAI is a learning which address students to solve problems given by the lecturer in small groups. This type of learning requires students to participate actively in the classroom. On the other hand, students are also taught to accept differences which might appear in the group. One of the efforts to overcome these problems is the cooperative learning model type TAI. The purpose of this study is to develop the students' understanding to communicate the concept of probability by using the cooperative learning model of TAI [6].

1.2 Problem

The problem of this study is: Is there any correlation between the implementation of cooperative learning model type Team Assisted Individualization and the improvement of the students’ mathematics communication ability?

1.3 Urgency of the Problem

The mathematics communication ability has not currently appeared within the students yet during the learning process. Thus, they tend to give up the tasks when they experience difficulties. This study is expected to be a reference and discourse for practitioners of mathematics education to understand more about the role of cooperative learning model type TAI in improving the mathematics communication ability.

2. Material and Method

2.1 Mathematics Communication Ability

In mathematics, communication plays an important role. Communication becomes an essential part of mathematics and mathematics education. [1], argue that communication is a process of delivering information, ideas, emotions, skills, and others through the use of symbols such as words, pictures, numbers, and others. Communication is the effort to deliver message, idea, or information from the communicator to the communicant and vice versa. Communication plays an important role in the learning process, including the mathematics learning. A broader sense of mathematics communication is presented by [8], namely: (a) connecting the concrete objects, figures, and diagrams into mathematical ideas; (b) explaining the ideas, situations and mathematical relationships orally or written by using concrete

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objects, pictures, graphs and algebraic expressions; (c) expressing daily life situations in mathematics language or symbol; (d) listening, discussing, and writing about mathematics; (e) reading with understanding a written mathematical presentation, making conjecture, making argument, formulating definitions and generalizations; (f) explaining and making inquiries about mathematics that have been studied.

2.2 Cooperative Learning type Team Assisted Individualization (TAI)

Cooperative learning model is a model of learning which emphasizes the use of students groups. The principle that should be upheld related to the cooperative groups is that every student is in a group should have the heterogenous level of ability (high, intermediate, and low) and if necessary, they must come from different races, cultures, and ethnic groups as well as considering the gender equality [8].

The cooperative learning type TAI has learning steps which foster the aspects of mathematics communication ability. The following steps are the steps of the cooperative learning modl type TAI:

Placement Test

In this step, the lecturer gives a pretest to the students. This method could also be replaced by observing the test result of the previous material or the average score that the students earned during certain duration of study. This step enables lecturer to figure out the students' weakness in particular topics.

Teams

This is recognized as an important step in the cooperative learning of TAI. In this step, the lecturer groups the students into some heterogenous groups consisting of 4-5 students each.

Teaching Group

The lecturer explains the material briefly before the tasks given.

Student Creative

The lecturer needs to emphasize and to create the students' perception that the individual success is determined by the success of their group.

Team Study

The students learn by using the students' worksheet. The lecturer also gives assistant to the students who need help individually. This step could also use the students who have good academic record to help the other members of the group as the peer tutor.

Fact test

The lecturer gives small test based on the facts got by the students, for instance, by delivering a quiz.

Team Score dan Team Recognition

The next step is that the lecturer gives score to the group's work and gives "salutation" reward towards the group which can pass the tasks well and also towards the group which still fails the tasks, for instance, by recognizing them as "the BEST group" or "the OUTSTANDING group", etc.

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Whole-Class Units

The final step is that the lecturer re-presents the material in the end of the chapter by using problem solving strategy for all the students in the classroom.

Each component of cooperative learning model type Team Assisted Individualization brings benefit to the lecturer, students, top groups and bottom groups who work together completing the academic tasks, namely: the clever students take the responsibility to help the weak group. Thus, they can develop their abilities and skills. The weak students will be assisted in understanding the subject matter, since there is no competition among the students because they work together to solve problems in dealing with different ways of thinking. Students do not only expect assistance from the lecturer, but also motivated to learn fast and accurately in all material. The lecturer could use only half of his/her teaching time so it will be easier to give individual assistance to the students [9].

2.3 Method

The method used in this study was a mixed method. The design was embedded concurrent design. In this mixed method, the quantitative method became the primary method while the qualitative method became the secondary method. The subject in this study was students of Statistics Study Program in one of state higher education institution in Makassar in the subject of Theory of Probability. The sampling technique used was purposive sampling. The sources of data in this study came from the students as research subjects. The instruments used were in the form of mathematics communication ability test, observation sheet, interview guide, documentation, and the researcher.

The data collection method used was the test (quantitative data) and the triangulation (qualitative data). The data analysis technique for quantitative data used correlation analysis, while the qualitative data were analyzed descriptively to support, clarify, and refine the results of the quantitative analysis in answering the problems. The test of mathematics communication ability has been validated so it is appropriate to be used in the study.

3 Results and Discussion

3.1 Quantitative Data Analysis

Before the correlation analysis is conducted, the score of mathematics communication ability should be tested by the normality test of *Kolmogorov-Smirnov* (*K-S*). The test was conducted by using *SPSS* 22.0 program. The result of the test is as follows.

Table 1. Test of Data Normality

Variable	Value of *K-S*	*Sig.*	H_0
Mathematics Communication Ability	0.096	0.06	Accepted

H_0: The data is normally distributted

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Based on the Table 1, we can see that the value of significance (*sig*.) for mathematics communication ability is more than 0.05. Hence, the null hypothesis is accepted. It means that the data is normally distributted. Therefore, the correlation testing should use the Spearman Rank Correlation to determine the correlaton coefficient and its significance value. By using the *SPSS* 22.0 program, it gives the following result.

Table 2. Correlation Testing and Its Significance

Correlation	r_{xy}	***Sig.* (2-*tailed*)**	H_0
Mathematics Communication Ability *Cooperative Learning TAI	0,421	0	Rejected

H_0: There is no correlation between the two variables

The Table 2 shows the value of significance (*sig*.) for the correlation is less than 0.05 so the null hypothesis is rejected. It means that there is a significance correlation between the Mathematics Communication Ability and the Cooperative Learning type TAI. Besides, the correlation coefficient gives positive result which means the correlation between the two variables is positive though the value is not high. Thus, the category of the correlation is medium [4].

Based on the explanation above, we can understand that the higher mathematics communication ability that the students have, the more score they get in the cooperative learning type TAI.

3.2 Qualitative Data Analysis

Based on the results of observation and interview towards some students, we found that the mathematics communication needs crative ideas. For instance, the following specimen is the problem of mathematics communication in the subject of Theory of Probability developed by the researcher.

Let the waiting time of a patient to get health service is denoted by T and having p.d.f gamma with $\alpha = k$ and $\beta = 1/\lambda$.

Determine the expectation value of the waiting time of the patient.

In order to solve the problem, student needs ideas to start. One of the alternatives is solving it directly. To start this method, student can use the definition of expectation value. The following is the sample of the solution process:

the expectation value of the waiting time of the patient is:

$$E(W) = \alpha\beta = k / \lambda$$

If k=1 ➔ then $E(W) = 1/\lambda$. It means that the waitng time for a patient is $1/\lambda$

The other sample is as follows:

Siti, a student of Statistics Study Program of Hasanuddin University, is doing a trial of throwing a balance coin 3 times, with the distribution function F(x) as follows:

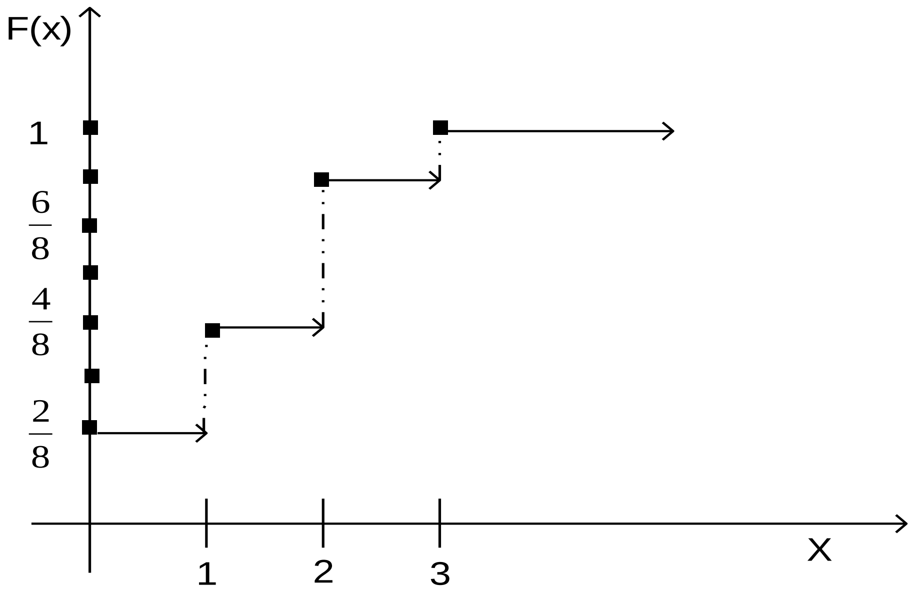
$$F(x) = \begin{cases} 0, & x < 0 \\ 1/8, & 0 \le x < 1 \\ 4/8, & 1 \le x < 2 \\ 7/8, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$$

Determine the cumulative distribution of the discrete variable and graph it.

Solution:

From the throwing of the balance coin 3 times:

$$F(x) = \begin{cases} 0, & x < 0 \\ 1/8, & 0 \le x < 1 \\ 4/8, & 1 \le x < 2 \\ 7/8, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}$$



The cumulative distribution discrete variable

In completing the two questions above, students discussed so that they engaged in a good mathematics communication to find or to understand the concepts. Each member of the group can work on a single problem as a form of shared responsibility. The implementation of cooperative learning model type Team Assisted Individualization emphasis moreon group awards, individual responsibility, and

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equal opportunities to share the results for each group member. Most of the students also admit that when they apply the learning model which emphasizes the utilization of student groups, the difficult problem will be easier to solve. This is because of the principles that must be upheld related to the cooperative groups that every student in a group must have the heterogeneous ability levels (high, intermediate, and low) and refers to a wide variety of teaching methods in which students work in small groups to help each other in learning the material. In addition, implementing cooperative learning type TAI requires the student to write down the steps in detail. Therefore, students need to understand the mathematics communication ability to understand the given problem.

4 Conclusion and Recommendation

4.1 Conclusion

The mathematics communication ability has a significant correlation with the cooperative learning type TAI, because in the class using the cooperative learning, the students are expected to help each other, to mutually discuss and argue, to sharpen the knowledge they have at the current time, and to overcome the gap in the understanding among students. Thus, the mathematics communication could be well done.

4.2 Recommendation

We suggest that the role of cooperative learning type TAI could be studied further related to the other mathematics abilities (cognitive aspect) and attitude (affective aspect)

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An Error Analysis of Students to Solve The First Order Differential Equations

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ABSTRACT

This study addresses to describe the errors of students to solve first order differential equations. This research is a quantitative descriptive were 28 students of mathematics education who enrolled differential equations course as subjects. In this study, each student solve three problems of first order differential equations involve the non-homogeneous, non-exact, and Bernoulli differential equations. Kiat framework was used to analyze the students error which consists of three criterias namely conceptual, procedural, and technical error. Based on the analysis of their tes results, find that approximately 40% students perform the procedural errors that are errors in determining the value of $aq - bp$ and manipulation the equations in solving non-homogenous differential equations. Furthermore, errors in determining the partial derivative of $M(x, y)$ and $N(x, y)$ and using logarithm and exponential numbers properties occurs in almost 70% students in solving non-exact differential equations. Finally, for Bernoulli type, more than 60% students perform conceptual erros in determining the general form and formula of Bernoulli differential equations. Moreover errors in solving the integration or determine of derivative always appear in the student's answer of each problems. This indicate a lack of ability of the students to understand the concept of integral and derivative.

Keyword: error analysis, solve, first order differential equations.

1. Introduction

Differential equation is a branch of mathematics that is widely used in applications of mathematics such as in engineering, science, and economics. In order to solve the problems associated with differential equations are required a good understanding of the concepts and procedures to solve the problems of differential calculus and integration. In other words, the understanding of differential calculus and integration will effect to the mastery of differential equations.

Studies to develop the students' ability in solving problems of differential equations has been done by many researchers. Camacho-Machin, et. al [2] analyzed and documented types of knowledge that university students exhibit to deal with basic knowledge that they had studied in a first ordinary differential equation course. Their research focus on three questions that are how do students interpret and deal with the concept of solution to an Ordinary Differential Equation (ODE)? To which extent do students use mathematical concepts they have previously studied to answer basic questions related to ODEs? And, to what extent do the students' answers privilege the use of certain type of representation to explore and examine issues related to ODEs? Results of their study show that the participants, in general, did not relate or use the meanings associated with the concept of derivative to make sense of questions or problems that involved differential equations concepts. Instead, they tried to identify or match a particular form of the involved equation with an algorithm to solve it. Rasmussen [5] offer a framework for interpreting students' understandings and difficulties with mathematical ideas central to new directions in differential equations. These new directions seek to guide students into a more interpretive mode of thinking and to enhance their ability to graphically and numerically analyze differential equations.

Studies about mathematical error have been conducted by many researcher with different categorization. Donaldson (cited by Orton [4]) described three types of errors, namely structural, arbitrary and executive. Structural errors were described as those "which arose from some failure to appreciate the relationships involved in the problem or to grasp some principle essential to solution". Arbitrary errors were said to be those in which the subject behaved arbitrarily and failed to take account of the constraints laid down in what was given. Executive errors were those which involved failure to carry out manipulations, though the principles involved may have been understood. Avital and Libeskind [1] categorized of student's difficulties encountered in mathematical induction into three types, namely conceptual, mathematical and technical difficulties. In this study, Kiat framework [3] was used to analyze the students error which consists of three criterias, namely conceptual, procedural, and technical error. *Conceptual* error refers to errors due to failure to grasp the concepts involved in the problem or errors that arise from failure to appreciate the relationships involved in the problem. *Procedural* error arise from failure to carry out manipulations or algorithms despite having understood the concepts behind the problem. The third type is *technical* error, refers to errors due to a lack of mathematical content knowledge in other topics or errors due to carelessness.

2. Method

This research is descriptive quantitative with twenty-eight mathematics education students who enrolled in differential equations as participants. All students were asked three problems of first order differential equations which involving of non-homogeneous, non-exact, and Bernoulli types of differential equations. The data was collected from students' answer of each problems. In this research, Kiat framework [3] was used to identify and analyze the errors perfomed of students in solving each problem. Then, the students answer were identified and classified to obtain the errors made by the students according Kiat framework. Furthermore, the percentage of each error performed by the students were determined.

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3. Result and Discuss

Problem 1: Non-Homogenous

Given the non-homogenous differential equations as follow,

$$(x - y - 3)\,dx - (x + y - 1)\,dy = 0. \qquad (1)$$

To solve problem 1, firstly, the equation (1) was arranged into the form $(ax + by + c)dx + (px + qy + r)\,dy = 0$. Secondly, determine the form of substitution that will be used in equation (1) by calculating $aq - bp$. If $aq - bp = 0$ then use $Ax + By = m\,(P(x) + Q(y))$ as a substitution form. Conversely, $x = u + h$ and $y = v + k$ were used as substitution form where h and k are obtained from the solution system of linear equation $ah + bk + c = 0$ and $ph + qk + r = 0$. Thirdly, substitute the substitution form obtained in second step into the equation (1) to gain homogeneous differential equation. Finally, the substitution $u = sv$ use to solve homogeneous differential equation derived in third step.

Based on the analysis of the student answers of problem 1, it can be identified the errors which carried out of the students in solving problem 1 as presented in Table 1.

Tabel 1. Students Errors of Problem 1

Types of Error	Percentage (Number of Students)	Description of Error
Procedural	35.7% (10)	1. Errors in determining the value of $aq - bp$ $aq - bp = 1(1) - (-1)1 = 2$. The correct answer is $1(-1) - (-1)(-1) = -2$ 2. Errors in manipulating the equation • $4 - k - 3 = 0 \Longrightarrow k = 1$. The correct answer is $k = -1$. • Substitution $u = Jv$ into $\ln(v) + \frac{1}{2}ln(J^2 - 2J - 1) = c$. The student's answer is $\ln(v) + \frac{1}{2}ln\left(\frac{u^2}{v^2} - 2v - 1\right) = c$. The correct answer is $\ln(v) + \frac{1}{2}ln\left(\frac{u^2}{v^2} - 2\frac{u}{v} - 1\right) = c$. • Substitution $x = u + 2$ and $y = v - 1$ into equation (1) obtained $(u - v - 1)du - (u + v)dv = 0$. The correct answer is $(u - v)du - (u + v)dv = 0$. • $(u - su)du - (u + su)(sdu + uds) = 0$. Student's answer is $(u + s^2u)du - (u^2 + su^2)ds = 0$. The correct answer is $(u - 2su - s^2u)du - (u^2 + su^2)ds = 0$.
Conceptual	7.1% (2)	1. Errors in determining the form of substitution for homogeneous differential equations 2. Student use unappropriate substitution to solve homogenous differential equations that is $u = s - v$. A correct substitution form is $u = sv$. 3. Unable to write the solution of non-homogeneous differential equations
Technical	17.8% (5)	1. Errors in solving the integration • $\frac{1}{2}\int \frac{k-1}{k^2-2k-1}dk = \frac{1}{2}\frac{2k-2}{k^2-2k-1}$ • $\frac{1}{2}\int \frac{2J-2}{J^2-2J-1}dk = \frac{1}{2}(J^2 - 2J - 1)$. 2. Errors in determining the derivative • Substitution form $v = su \Longrightarrow dv = sds + uds$. The correct answer is $dv = sdv + uds$. • Substitution form $u = sv \Longrightarrow du = sdv - vds$. The correct operation is +. 3. Do not replace dx and dy with du and dv after the substitution. Substitution $x = u + 2$ and $y = v - 1$ into equation (1) obtained $(u - v)dx - (u + v)dy = 0$.

Table 1 shows that there are eleven (approximately 40%) students perform procedural errors in determining the value of $aq - bp$ and manipulation of the equation. It seems that they do not arrange in advance the equation (1) into the basic form of non-homogeneous differential equations, namely $(ax+by+c)dx+(px+qy+r)dy = 0$. However, they directly used the value of a, b, q, and p from equation (1). Furthermore, the manipulation errors due to less carefulness of students in the calculation.

Problem 2: Non-Exact

Given the non-exact differential equations as follow,

$$(x+ y)\,dx + x\ln x\ dy = 0. \qquad (2)$$

From equation (2), it can be defined $M(x,y) = x+y$ and $N(x,y) = x\ln x$. The equation (2) called non-exact differential equation when $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$. To solve the equation (2), it must be changed into exact differential equation by multiplying it with the integral factor $U = e^{\int \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}dx}$. The results of errors analysis of the student's answers are presented in Table 2.

Tabel 2. Students Errors of Problem 2

Types of Error	Percentage (Number of Students)	Description of Error
Procedural	10.7% (3)	Mistakes perform of arithmetic operations • $x^{-1}(x+y)dx + x^{-1}(x\ln(x))dy = x^{-1}ydx + \ln x\ dy$ • $1-(1+\ln x) = \ln x$.
Conceptual	3.6% (1)	Wrong for determining the integral factor. The students write $\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}$ instead of $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}$.
Technical	67.9% (19)	1. Errors in determining the derivative of $M(x,y) = x+y$ and $N(x,y) = x\ln x$. The student's answer are $\frac{\partial}{\partial y}(x+y) = x+1$, $\quad \frac{\partial}{\partial x}(x\ln x) = \frac{1}{x}$ $\frac{\partial}{\partial y}(x+y) = x$, $\quad \frac{\partial}{\partial x}(x\ln x) = \frac{1}{x^2}$ $\frac{\partial}{\partial x}(x\ln x) = -\frac{1}{x^2}$, $\quad \frac{\partial}{\partial x}(x\ln x) = \ln x - 1$ 2. Errors using logarithm and exponential numbers properties $e^{-\ln x} = -x$

Table 2 shows that about nineteen (almost 70%) students commit the technical errors especially in determining the partial derivative of $M(x,y)$ and $N(x,y)$. The correct answer of the partial derivative $M(x,y)$ and $N(x,y)$ are $\frac{\partial M}{\partial y} = 1$ and $\frac{\partial N}{\partial x} = 1+\ln x$. It is indicate that most of the students have a lack understanding of partial derivatives. Table 2 also exhibit that almost all of the students understand the procedure to determine an exact differential equation.

Problem 3: Bernoulli

Given the Bernoulli differential equations as follow

$$\frac{dy}{dx} = xy\,(1-x^2y). \qquad (3)$$

The equation (3) have to arranged into general form of Bernoulli differential equations that is $\frac{dy}{dx} + P(x)y = Q(x)y^n$, $n \neq 0$. To solve a new form of equation

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obtained was used the general formula solution of Bernoulli differential equations that is

$$y^{(1-n)} = e^{-\int (1-n)P(x)\,dx}\left\{\int e^{\int (1-n)P(x)dx}(1-n)Q(x)dx + c\right\}. \quad (4)$$

A summary of the errors analysis of the student's answers of problem 3 are presented in Table 3.

Tabel 3. Students Errors of Problem 3

Types of Error	Percentage (Number of Students)	Description of Error
Conceptual	60.7% (17)	Errors to determine the general form of Bernoulli differential equations $\frac{dy}{dx} - (1 - x^2y) = 0$ $\frac{dy}{dx} + x^3y^2 = xy$ $\frac{dy}{dx} + x^3y^2 = 0$ $dy = (xy - x^3y^2)dx$ $\frac{dy}{dx} + x^3y^2 = xy$ $\frac{dy}{dx} - xy + x^3y^2 = 0$ $\frac{dy}{dx} - xy = 0$ $ydy = (x - x^3y)dx$ $\frac{dy}{dx} - xy + x^3y^2 = 0$ $y^{-2}\frac{dy}{dx} - xy^{-1} = x^3$ Errors to determine the general formula solution of Bernoulli differential equations $\left\{\frac{dy}{dx} - xy + x^3y^2 = 0\right\}\frac{dx}{dy}$
Technical	17.9% (5)	1. Errors in solving the integration $\int e^{\frac{1}{2}x^2}x^3dx = 2e^{\frac{1}{2}x^2}\frac{1}{4}x^4 + c$ $\int e^{\frac{1}{2}x^2}x^3dx = \frac{1}{4}e^{\frac{1}{2}x^2}x^4 + c$ $\int e^{\frac{1}{2}x^2}x^3dx = e^{\frac{1}{2}x^2}x^2 + c$ Can't solve $\int e^{\frac{1}{2}x^2}x^3dx$

Table 3 exhibit that most of the students (more than 60%) perform a conceptual errors in solving equation (3). They have failed to manipulate the equation (3) into the general form of Bernoulli differential equations. One of them failed to determine the general formula solution to solve the equation. He multiply the general form by $\frac{dx}{dy}$ instead of use the equation (4). As well as on problem 1 and 2, the students also carry out an errors in solving the integration. Further, there are five students who did not answer the problem 3.

Errors in solving the integration or determine of derivative always appear in the student's answer. The students seemed lack of ability the concept of integral and derivative. In problem 2, most of the students lack in understanding concept of partial derivative. Therefore they can not determine the partial derivative forms for problem 2 correctly. Kiat [3], in his study regarding students difficulties in solving integration, found that the students focus more on the procedural aspects of integration than on the conceptual aspects. Consequently the students lacked in conceptual understanding of integration. Sanchez [6] have observed certain errors that students made when solving an ordinary differential equation. Those errors are calculating the integral of a product of functions as the product of the integrals and does not recognize the type of ordinary differential equation correctly. Consequently, he made a mistake in the solving process. The result of this study coincide with Sanchez [6] conclusion, especially in problem 3, which most of the students seemed confused to determine type of differential equations to solve the given equations.

4. Conclusion

Students Errors in solving first order differential are very diverse. For non-homogenous differential equations, approximately 40% students perform the procedural errors in determining the value of aq-bp and manipulation the equations. Furthermore, errors in determining of partial derivative M(x,y) and N(x,y) and using logarithm and exponential numbers properties occurs in almost 70% students in solving non-exact differential equations. Finally, for Bernoulli type, more than 60% students perform conceptual erros in determining the general form and formula solution of Bernoulli differential equations.

These results provide an overview of the errors performed by the student in solving first order differential equations. However, this study is limited to the analysis of the documents on the student's answers in solving the problems given. Further analyzes through interview will allow obtaining more complete information about the cause of the errors made by the students.

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ALTERNATIVE COMPLETION OF POVERTY IN INDONESIA THROUGH *MUDHARABAH*

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ABSTRACT

Islamic bank is the bank which operates using 0% interest. One of the principles applied in it is the principle of *Mudharabah* (profit sharing), which is cooperation or contract for business between two parties in which the first party (Islamic bank) provides all/ 100% of the capital, whereas other administering the capital. In Indonesia, there are still over 28 million poor people or 11.25% of the total population. Profit sharing of Islamic bank could be a major capital and alternative completion in accomplishing poverty in Indonesia, particularly for it is espoused by its large amount of assets. This study used the data published by Bank Indonesia in 2014, at which the total assets of Islamic bank is 255.2 billion rupiah with the composition of assets used to finance *Mudharabah* is 14.4%. There are two concepts of poverty completion by using *Mudharabah* Principle in this study, namely the concept of provision of venture capital collectively and individually. From the analysis results, generated that the concept of provision of venture capital collectively takes 38 years to complete poverty in Indonesia, while the concept of provision of venture capital individually takes 16 years.

Keyword: Islamic Bank, Mudharabah, Poverty Completion.

1. Introduction

Indonesia is one of the world's largest populations. Based on the data released by Statistics Indonesia or known as BPS, the total number of Indonesian Population in 2015 is exceeding250 million [1]. It's definitely a fact that, with that large number of population, Indonesia hasn't been capable of escaping from poverty problem. Despite the rank of poverty descended in 2013 to 2014, but the rank of poverty in Indonesia is still in top chart of poverty in the world. Statistics Indonesia recorded that over 28 or 11.25% of the total population are living in poor line [2].

The magnitude of poverty rate in Indonesia cannot be regardless of the lack of work field, in addition to many communities do not have the capital to be able to build their own business. Although, in fact, there are so many financial institutions that provide loans as startup capital, but the interest rate is high enough. Thus, nevertheless, they will think twice of borrowing, notably with the risk that will be borne if the loan cannot be repaid due to the business bankrupt.

In this case, with the existence of Islamic banks, namely banks operating with 0% interest, paving the way to remove the obstacle and dilemma for the community in terms of lending business capital without having to fear the above risk. For, besides as channeling funds, the application of 0% interest on any loaning shows the critical role of Islamic banks in evolving economies the middle class society and open up new work fields.

One of the principles of Islamic banks which is suitable to drive thereal sector of economy in Indonesia is Mudharabah, namely which is cooperation or contract for business between two parties in which the first party (Islamic bank) provides all/ 100% of the capital, whereas other (people) administering the capital.Mudarabah business profit sare divide daccording to the agreement set for thin the contract, where as if the loss ensues, then it will be borne by the capital owner for the loss was not due to negligence of the capital administrator. Yet, for fraudor negligence of the administrator, then he should be responsible for such losses

Mudarabah business profit sare divide daccording to the agreement set for thin the contract. If the loss ensues, then it will be borne by the capital owner (Islamic Bank) for the loss was not due to negligence of the capital administrator. Yet, for fraudor negligence caused by the administrator, then he should be responsible for the losses.[3].

Therefore, from the prior statements, the Author wishes to adopt a paper title"Alternative Completion of poverty in Indonesia through Mudharabah".

2. Material and Method

Islamic bank is a bank founded by a group of Muslims, with its main characteristicis "no interest", or commonly called "profit sharing bank". The pioneer of such bank was Islamic Development Bank(IDB).[4] Veithzeel said that Islamic bank has two significant roles for economic grow thin Indonesia, they are: as business entity and charities. As business entity, Islamic bank has purposes, namely as an investment manager, as an investor, and as service provider.Where as, as charities, Islamic bank has purpose as social fund manager.[5].

With these two roles, it is not surprising that Islamic bank is able to survive in conditions of economic crisis in which conventional bank suffer negative spread in its business. Thus, this exhibits the existence of Islamic banking as a system that is resilient and able to encourage economic growth in developing countries.

This important role, certainly, is supported by *Mudharabah* principle adopted by Islamic bank. With the existence of this contract, it will aid those who have capital and do not have the business skills to develop their funds. On the other hand, the administrators who own business skills, be able to run their business becauseof acquiringthe capital gain from the capital owner. *Mudharabah*, hence, can be used as an investment instrument and to actuate the real sector of economy.

Mudarabah business profits are divided according to the agreement set forth in the contract, in which the results obtained by each party must be dependent on profit sharing factor. This profit sharing is measured using the principle of "benefit ratio". According to Adiwarman, this ratio reflects the wage received by both parties

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doing *Mudharabah*. The figureof this ratio varies,it could be 50:50, 60:40, 70:30, 80:20, and even 99: 1. But the Jurists agreed that the ratio of 100: 0 is not allowed.[6]

To further concretizeabout this benefit ratio, the Author imparts an example ofcalculation of benefit ratio in a financing by an Islamic bank to certain business: Ny. Pariani wants to do business with a capital of 50,000,000.00 IDR. It is estimated that, of these efforts will earn 10,000,000.00 IDR per month and the capital fully provided by Bank SyariahManggar. From that earning, it's firstly separated out for restoring the capital, for instance, 4,000,000.00 IDR. That means, the remaining revenues is 6,000,000.00 IDR. This balance is what will be distributed to both parties, Bank SyariahManggar and the capital administrator in accordance with the previous agreement, i.e. as instance 60: 40. Thus, obtained the profit income as below:

For Bank SyariahManggar = 60% x 6,000,000.00 IDR = 3,600,000.00 IDR

ForNy. Pariani = 40% x 6,000,000.00 IDR= 2,400,000.00 IDR

The data sources of this research is secondary data, i.e.the documentation of Bank Indonesia in the form of data regarding to the assets of Islamic banks in Indonesia until 2013, where the total assets of Islamic banks is 233.13 billion rupiah with the composition of assets used to finance *Mudharabah*is 14.4%[7], as well as the documentation Statistics Indonesia in the form the total of poor people to 2014.

The method used to analyze completion of poverty through *Mudharabah* done by harnessing both data sources above, and some mathematical formulas, namely percent formula (%) to calculate the nominal of *Mudharabah*financing and poverty descent per year, as well as the aid of multiplication and division formulas. The steps are as follows:

1. Determining the total assets of Islamic banks to finance *Mudharabah* per business
2. Calculating the nominal of *Mudharabah* financing, with the formula:

   Nominal forMudharabah = Composition of Assets for Mudharabah x the Total Assets
3. Analyzing the time needed to complete poverty in Indonesia, in this case the Author used two concepts:
   a. Provision of capital for business collectively
   b. Provision of capital for business individually

3. Result and Discussion

Poverty has been the most frightening scourge for every nation, inasmuch as it could be a major agent for appearing social inequality and social disaster, such as theft, murder, robbery, starvation, malnutrition, etc, and this can be clearly seenin developing countries such as Indonesia even then. Therefore, there needs an alternative completion which is capable of settlingpoverty, also as the first step in eroding social gaps caused by it.

Islamic bank as capital provider for those who want to build and expand their business, particularly those living in poverty, is an institution that should take the leading role and the first step in fulfilling poverty, since it offers the concept of

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provision of capital that has gone through an agreement between the bank with itscustomers so that no single party will feel aggrieved then, with the calculation of profit through profit sharing principle that has also been agreed its magnitude for each party. Analysis of the use of this principle for poverty completion in Indonesia is as follows:

The Assets of Islamic banks till is 233.13 trillion, 14.4% of which is used for Mudharabah financing per year. Then the nominal is:

Nominal fo rMudharabah = 14.4% x 233.13 trillion
= 33,570,720,000,000.00IDR per year

Furthermore, there are two choices of how to reduce poverty in Indonesia by using the profit sharing concept of *Mudharabah*, namely provision of capital for collective business and business individually.

1. Provision of Capital for Collective Business

For example, the above fund for *Mudharabah* financing is fully disbursed to be business capital for the poor in Indonesia. Then, the average of capital given to each customer group is 500,000,000.00 IDR. So every year, the number of people who will get financing for theirbusiness capital is:

$$= \frac{\text{Nominal for Mudharabah}}{\textit{Capital Given to every Customer}}$$

$$= \frac{33{,}570{,}720{,}000{,}000}{500{,}000{,}000}$$

$\approx$67,141 customer groups

With the availability of many facilities to open a business and market business products, where every customer group consists of 11 people (one person as chairman and others are members). Thus, every year there will be decline of the number of poor people:

$$= 67{,}141 \times 11 = 738{,}551 \text{ people}$$

Thus, every year the reduction of poor people in Indonesiais738,551 people, or 2.6%. Therefore, the time required to resolve poverty in Indonesia by using the concept of provision of capital for collective business is:

$$= \frac{28{,}280{,}010}{738{,}551}$$

$$= \frac{100\%}{2.6\%}$$

$\approx$ 32 years

So, with this concept, Indonesia takes over 38 years to complete the poverty.

2. Provision of Capital for Individual Business

For example, the above fund for *Mudharabah* financing is fully disbursed to be business capital for the poor in Indonesia. Then, the average of capital given to each customer individually is 20,000,000.00 IDR. So every year, the number of people who will get financing for their business capital is:

$$= \frac{\text{Nominal for Mudharabah}}{\textit{Capital Given to every Customer}}$$

$$= \frac{33{,}570{,}720{,}000{,}000}{20{,}000{,}000}$$

$$\approx 1{,}678{,}536 \text{ customers}$$

So, every year the number of poor people who are accommodated in work field is 1,678,536.

Thus, every year the reduction of poor people in Indonesia is 1.678.536 people, or 5,94%. Therefore, the time required to resolve poverty in Indonesia by using the concept of provision of capital for individual business is:

$$= \frac{28{,}280{,}010}{1{,}678{,}536}$$
$$= \frac{100\%}{5.94\%}$$
$$\approx 16 \text{ years}$$

So, with this concept, Indonesia takes over 16 years to complete the poverty.

4. Conclusion

Based on the prior exposures, it can be shortenedthat *Mudharabah* is principle applied by Islamic Bank in terms of providing the business capital for people, in which the profit obtained by two parties, Islamic Bank and people (capital administrator), is measured using benefit ratio that has agreed by both parties in the first place. Thus, this can be the major thing in accomplish poverty problem in Indonesia.From the analysis results, by the aids of the data in the form of assets of Islamic Bank and the total number of poor people in Indonesia as well as some mathematical formulas,generated that the time needed to complete poverty by using the concept of provision of capital for collective business is 32 years, while the concept of provision of capital for individual business is 16 years.

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Using Daily Problems to Measure Math Literacy and Characterise Mathematical Abilities for Students in South Sulawesi

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ABSTRACT

There are many reasons why math literacy is important for pupils in the school and daily problems are more realistic to understand what kind of mathematical abilities they have. This study aims to describe the use of daily problems as an indicator to measure mathematical literacy and to characterise mathematical abilities of students. Additionally, mathematics literacy is important to support mathematical abilities as the form of competence of learners in mathematics. Survey research was conducted to 159 junior high school students in South Sulawesi. This research applied mathematical literacy test to find data of students' literacy. The results showed that the tendency of the percentage of students to be able to give a reason but it is not accurate to not give a reason at all starting from 32.08% to 96.86%. This figure represents the students' ability to argue using their mathematical knowledge is still below 50%. In other words, most of the capabilities of mathematical knowledge of students is not well developed due to the mathematical literacy of students is not developed.

Keyword: Daily Problems, Math Literacy, Mathematical Abilities.

1. Introduction

Literacy is often associated with letters (characters) or linguistically interpreted as "literate". Literacy has been a warm conversation among international and became the hope of the world. Various parties heavily literacy assessment and find ways to improve literacy in many countries. They have realized that the world to face the younger generation getting older necessary perceptive and critical. However, Indonesia in the international level is seen has not been able to give birth to a literated generation. According to data from the Programme for International Student Assessment (PISA) 2000, 2003, 2006, 2009 and 2012, Indonesian students' mathematical literacy is still low. In 2012, Indonesia was ranked 64 out of 65 countries, with a score of 375, while international score is 494 [3].

For elementary level, there are seven of standard of competencies that students should be engaged with. From those, every single item were addressed to

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how they can apply for their daily life [6]. Compared with revised curriculum (K-13), the target of knowledge domain is facilitating students having factual and conceptual knowledge based on their curiosity about science, technology, art, and culture within humanity perception, nationality, citizenship, and civilization in related with phenomena and events in the environment, school, and play park. Meaning that the important knowledge they have after learning is addressed to tackle any kind of problems students face in daily activities. However, this is not in line with the data above which is Indonesian students below the level of PISA standard. It means that there is such students' activities incompatible with program to support math literacy or any program to deconstruct students' literacy. Because of this, [5] argued that the instrument of learning assessment designed by teachers of Junior Secondary School in Indonesia generally presented lack of literacy development.

PISA mathematical literacy within the framework of Mathematics in 2012 defined as an individual's ability to formulate, use and interpret mathematics in various contexts. Including the ability to perform reasoning mathematically and using the concepts, procedures, facts, as a tool for describe, explain and predict a phenomenon or event. Mathematical literacy can help individuals to recognize the role of mathematics in the real world and as a basis for consideration and determination of decisions needed by society [2]. Being matematically literate, competencies needed that form the heart math literacy [4], for example PISA.

Harnessing more useful help of PISA problems, students can be triggered to exhibit their mathematical ability. [1] supposed that every student has the ability to study mathematics, even though some students explore and create connections faster than others. All of students have some mathematical ability, but some of them have potential far beyond what dominant people are provided to trust. Furthermore, Borovik & Gardner emphasise indicators of mathematical abilites including ability to make and use generalisations, rapid and sound memorisation of mathematical material, ability to concentrate on mathematics for long periods, ability to offer and use multiple representations of the same mathematical object, an instinctive tendency to approach a problem in different ways, ability to utilise analogies and make connections, preparedness to link two (or more) elementary procedures to construct a solution, ability to recognise what it means to "know for certain", ability to detect unstated assumptions in a problem, and either to explicate and utilise them, or to reject the problem as ill-defined; a distinctive tendency for "economy of thought," striving to find the most economical ways to solve problems, for clarity and simplicity in a solution; instinctive awareness of the presence and importance of an underlying structure; lack of fear of "being lost" and having to struggle to find one's way through the problem, a tendency to rapid abbreviation, compression or a curtailment of reasoning in problem solving; an easy grasp of encapsulation and de-encapsulation of mathematical objects and procedures.

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Mathematical literacy involves seven basic skills that should be possessed [2], namely: (1) Communication, ability to communicate problems; (2) mathematizing, the ability to change the problems of the real world into the form of mathematics or vice versa; (3) Representation, the ability to restate a mathematical problem; (4) Reasoning and Argument, the ability to reason and reason; (5) Devising Strategies for Solving Problems, the ability to use problem solving strategies; (6) Using Symbolic, Formal and Technical Language and Operation, capability uses symbolic language, formal language and technical language; (7) Using Mathematics Tools, the ability to use mathematical tools, for example in the measurement.

Combining those two theories, this study only emphasize what is the number of students who have good understanding as well as powerful reason in their constructed answer. Following this, what kind of abilities are in which students are lack of mathematical knowledge.

Based on this, it is interesting to bring mathematics problems, in particularly daily problems, as challenging topic to measure mathematical literacy. Learning from problems literacy on PISA and TIMSS (Trends International Mathematics and Science Study) which requires reasoning and problem solving skill share experience to develop local context as the main idea of the problems. Subsequently, daily problems in this case refers to what students have understanding on it. Meaning that there is no strange context used to explore their such literacy or the things that students have no idea related with.

Taking advantage in classroom experience, it was undoubtful that many of secondary graders can show their works to calculate and compute as mathematical process in solving the problems. Even word problems created by their teachers are not getting difficult for children to solve it using their arithmetic understanding. However, the solution include the correct procedure it becomes questionable. The challenge is interconnection between the problem and its solution. If the readers try to understand step by step of arithmetic procedure, for example, they will find no problem on it. The thing is when connecting the main idea of problem with the solution since students only consider about the numbers, separated with the main problem. We called this obstacle is low interpretation. Lastly, how to communicate by showing arguments or reasons to accompany their answer is not easy to find students' work excellently, especially for South Sulawesi students.

Starting from this problem, this study has endeavor to share information how good is South Sulawesi students to develop their mathematical literacy in solving mathematical daily problems and what kind of mathematical ability they exhibit in their performance.

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2. Methodology

Survey research was type of exploration to support this study conducted to 159 junior high school students in South Sulawesi. This research applied mathematical literacy test to find data of students' literacy. Then, the rubric of literacy assessment was provided for checking students' answer to measure their literacy. It was also accompanied by indicators checking of mathematical abilties.

In general, the survey carried out in several stages, namely: 1) Determine the research problem which are math literacy and mathematical abilities; 2) Making survey design, this application refers to measure math literacy by giving test all of respondents and the result would be identified what kind of mathematical abilities appear; 3) Develop a survey instrument, since there are two kinds of target of this study, the two instruments are including essay test and rubric as the guide to measure literacy and math abilites; 4) Determine the sample, this study was conducted in South Sulawesi; meaning that all of students who study in this area became main focus to explore the target; 5) Data processing and analysis; 6) Interpretation of the data; and 7) Making conclusions and recommendations.

In particularly, essay test was constructed here including 15 items. Its content are divided into three kinds of cognitive ability which are 7 items of analysis problems, 2 items of comprehension problems, and 6 items of application problems.

After collecting data, it was analyzed descriptively to determine the result of literacy math measure. Of course, quantitatively the data was analyzed using descriptive statistics, but in this case graph and diagram are used to describe the distribution of math literacy score. Meanwhile, the percentage is also exhibited to draw in terms of math literacy and mathematical abilities.

3. Result and Discussion

Of 15 items of math problems, there are 7 out of analysis items, while the rest of those are divided into 2 numbers of comprehension, and 6 numbers of application. These problems were developed using daily human activities. Started such daily problems, students are expected to explore their idea and trigger them to construct mathematical idea based on the situation. So, the test would be interesting and not exhaust thinking process in which students push themselves to recall mathematical formula or memorized knowledge.

Following this, there are 4 mathematical abilities which students will perform related with problems available. Firstly, ability to offer and use representation of mathematical object including reading or constructing graph, picture, table, or diagram. Then, abilty to utilise analogies and make connections, and preparedness to link two (or more) elementary procedures to construct a solution to a multi-step problem. Lastly, an instinctive tendency to approach a problem in different ways:

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even if a problem has been already solved, a child is keen to find a alternative solution.

After trying out of competition, what we have it is shown in the following diagram. In order to understand what kind of information that this diagram is going to share us, mathematical literacy can help individuals to recognize the role of mathematics in the real world. Meaning that individual's ability to formulate, use and interpret mathematics in various contexts are abilities to perform reasoning mathematically and using the concepts, procedures, facts, as a tool for describe, explain and predict a phenomenon or event. So, the main point what this study want to explore how good is South Sulawesi students to develop their mathematical literacy in solving mathematical daily problems.

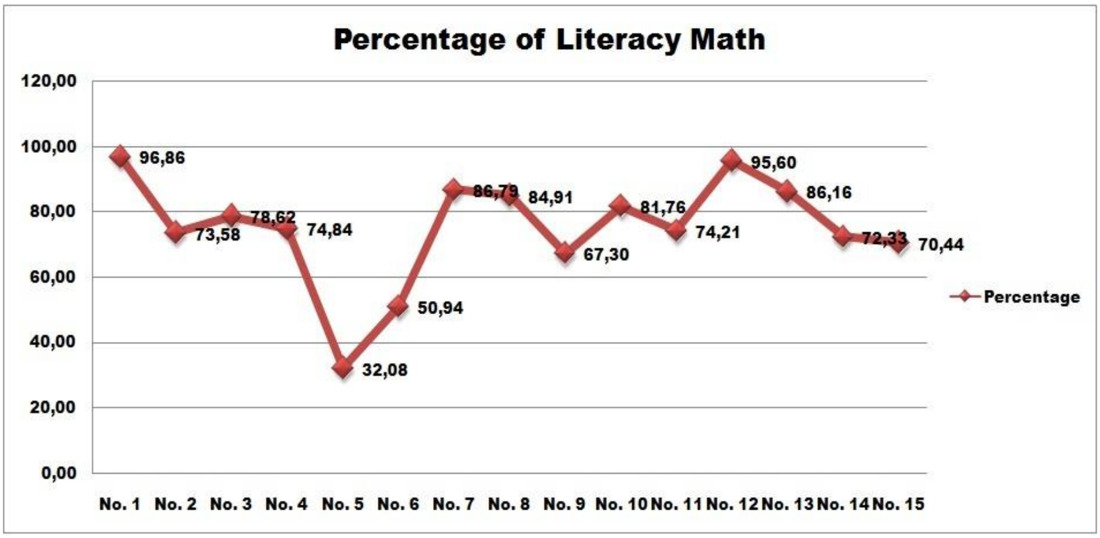

Figure 1. Percentage of Literacy Math Score

This figure shows that the score given to students' answer including mathematical concepts and explanation to connect between the formula and the context. Shortly, there are two considerations here. Firstly, students answer the question using mathematics formula correctly, and the last is promoting reasonable explanation to accompany the mathematics concept. This bring students to to recognize the role of mathematics in the real world and as a basis for consideration and determination of decisions needed by society [2].

The red line above describes the tendency of the percentage of students to be able to give a reason but it is not accurate to not give a reason at all starting from 32.08% to 96.86%. In detail, problem number 5 becomes the least of literacy percentage that means more than a half of students giving reasonable answer. The rest of items are quantified higher than 50%. This emphasize that students of secondary school in South Sulawesi need to be trained in their daily activities

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constructing reasonable explanation, understanding the context, and making connection between the mathematical formula and the context of problem.

Table 1. Distribution of Mathematical Abilities Based on the Test Item

	Mathematical Abilities							
	Item Number	MA 1	Item Number	MA 2	Item Number	MA 3	Item Number	MA 2 & 4
Percentage	No. 1	3,14	No. 4	25,16	No. 12	4,40	No. 6	50,94
	No. 13	13,84	No. 5	67,92	No. 7	13,21		
	No. 8	15,09			No. 10	18,24		
	No. 3	21,38			No. 9	32,70		
	No. 11	25,79						
	No. 2	26,42						
	No. 14	27,67						
	No. 15	29,56						

Remark:
MA1 : ability to offer and use representation of mathematical object including reading or constructing graph, picture, table, or diagram
MA2 : abilty to utilise analogies and make connections
MA3 : preparedness to link two (or more) elementary procedures to construct a solution to a multi-step problem
MA4 : an instinctive tendency to approach a problem in different ways: even if a problem has been already solved, a child is keen to find a alternative solution.

The table 1 above explain that no more than a half of students have well-developed ability to share idea using mathematical object such as graph, picture, table, or diagram. As we know the object of mathematics should be understood as the package knowledge in order to construct and wrap their explanation mathematically. Using graph or diagram is one of skills that students can not show their work effectively. This also is followed by the ability of constructing link two (or more) elementary procedures to construct a solution to a multi-step problem. The highest percentage of this ability is 32,70% and holds only for item 9, whereas the rest is below 20%.

Although it cannot be generalized that students are not nicely done to show their analogies and make connection between context and mathematical knowledge, the no. 5 is little better than that no 4. Meaning that students are being more familiar with problem no. 5 than that problem 4. Considering the content of problems, MA1 is related with MA 2 in which students are low ability to solve problems considering representation mathematical object using table, chart, so on. So, this bring us to find the cause of the percentage of no 5 (67,92%) leading for the percentage of no. 4 (25,16%).

Unfortunately, only no. 6 which was constructed to explore combination between MA2 and MA4 (the abilty to utilise analogies and make connections and approach a problem in different ways to find a alternative solution) is produced as one of items. The percentage of this ability is more than 50%, and it is better than that both of MA1 and MA3. Based on the context, problem no. 6 is quite similar with problem no. 5 talking about solid figure. It means that students' knowledge are being developed if topics are related with solid figure construction, in particular cube.

Overall, the mathematical abilities of students to argue using their mathematical knowledge is still below 50%. In other words, most of the capabilities of mathematical knowledge of students is not well-developed due to the mathematical literacy of students is not developed.

4. Conclusion

Using 15 items of daily mathematical problems, this study shows that mathematical literacy of students in South Sulawesi is not well-developed in order to construct their answers. The fact that this study obtained is the percentage of students to be able to give a reason but it is not accurate to not give a reason at all starting from 32.08% to 96.86%. This emphasize that students of secondary school in South Sulawesi need to be trained in their daily activities constructing reasonable explanation, understanding the context, and making connection between the mathematical formula and the context of problem.

This study also explored four mathematical abilities which students will perform related with problems available. Firstly, ability to offer and use representation of mathematical object including reading or constructing graph, picture, table, or diagram. Then, abilty to utilise analogies and make connections, and preparedness to link two (or more) elementary procedures to construct a solution to a multi-step problem. Lastly, an instinctive tendency to approach a problem in different ways: even if a problem has been already solved, a child is keen to find a alternative solution. The result is the mathematical abilities of students to argue using their mathematical knowledge is still below 50%. In other words, most of the capabilities

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of mathematical knowledge of students is not well-developed due to the mathematical literacy of students is not developed.

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Developing A Mathematics Instructional Model Based on RAKIR (Child Friendly, Innovative , Creative and Realistics) At Junior High School

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ABSTRACT

The aim of this study is to develop a mathematics instructional model based on RAKIR (Child Friendly, Creative, Innovative and Realistics) in Junior High School. This study is a development research, conducted in the form of development model proposed by [7]. It consists of four development phases: (1) the preliminary investigation phase conducted to get information about mathematics instructional model theories, learning theories and the analysis of the mathematics in Junior High School, (2) the design phase conducted to design the instructional model based on RAKIR, (3) the realization phase conducted to develop mathematics instructional model by following the design phase and (4) the revision, evaluation and test phase of the mathematics instructional model prototype being developed and was validated by an expert in learning mathematics and try out. This research was conducted at the Junior High School Muhammadiyah Program Khusus Surakarta. These result of this study shows that: (1) the learning management based on RAKIR by teacher was in " very good" category with score 3,50 , (2) the prototype model had improved students' activities were in "very good" category and (3) the students' responses toward the mathematics instructional model was included in "positive" category. Besides that, it was obtained the instructional mathematics model based on RAKIR with the following syntaxes: (1) explaining the learning objectives and motivating students, (2) giving the contextual problems that are familiar with students, (3) processing mathematics' abstraction (mathematics vertically), (4) formulating the solving strategies, (5) communicating the result of discussion and (6) giving the inferences of mathematics material.

Keywords : child friendly, innovative, creative and realistic.

1. Introduction

Education is a conscious effort of man to guide humans in order to develop the personality and ability in accordance with the values that prevail in society. Every human being in need of education, until when and wherever he's been. Education in school can not be removed from the process of learning and interaction between teachers and students. [2] giving definition of learning is a permanent change in response highly potentiality which accurs as a result of reinforced practice. From this definition, it means that learning can produce behavior change relatively permanent

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students and teachers as doer change. Thus, learning is the assistance given by teachers in order can occur the process the acquisition of science and knowledge, mastery of skills and behaviour, as well as the formation of attitudes on students. In other words, learning is a process to help students to learn well. Thus, the teacher is a spearhead for feeding children contributed. The magnitude of the responsibility, then the Government gives awards for teachers by placing teachers as professionals who poured in to law teachers and professors in 2005. Teachers as professionals must have the four competencies that are teaching competence, professional competence, social competence and personality competence.

However, [11] states based on the results of a survey conducted by the Directorate General PMPTK informed that most of the teachers have a low competence and most of them have not been trained on the pedagogic. This condition is in line with research conducted [4] which informs the low ability of teachers in the assessment of learning outcomes and learning management. This means there are still many teachers that have low competence in teaching and learning. Because of that , they still dominate in teaching a class, do not involve students actively in the learning. They still adhere to pragmatism is everything, teachers consider the students as an object not as the subject. So with this condition, it resulted in a low level of activity and creativity of students in learning.

On the other hand, the results of research conducted by [10] said that students need to have (1) the ability related to mathematics that can be used in solving mathematical problems, another lesson, or problems related to real life; (2) the ability to use mathematics as a tool of communication; and (3) the ability to use mathematics as a way of reasoning that can be used in any circumstances, such as critical thinking, logical thinking and systematic thinking. [6] and [9] also conclude in their research that in learning mathematics liveliness, creativity and communication of mathematics students are urgently needed to improve the understanding of mathematical concepts. This will not be easily filled by students, if it is not supported the ability of teachers in teaching as well as learning resources . Learning resource limitations either literature or media for learning mathematics is one of the obstacles during the process of learning. In addition to these problems and supported teachers' ability in learning conditions are also very poor, it is necessary to find solutions to solve those problems.

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From the above explanation , then it needs to be developed the mathematics instructional model based on RAKIR (Child Friendly, Innovative , Creative and Realistics). This research needs to be done in order to support the implementation of the curriculum of 2013 and also to increase students' activity and to improve the creativity of teachers in the teaching and learning of mathematics.

2. Materials and Method

Child friendly learning refers to the the condition of the school/class that is a friendly place for students to learn. Friendly defined as a condition that is safe, joyful, and free. Safe refers to a condition that is free of violence and arbitrariness. Joyful is a condition of class that make students learn with pleasure. Free refers to the freedom of students to speak their opinion. According to [8] said that the inclusion of mathematical activities in the museum, science center and other informal inveroments has the potential to complement formal learning in school mathematics classrooms, promote possitive attitudes to ward of the mathematics. This means the learning process does not have to occur in the classroom, but it can be done outside the classroom even outside of school. Thus, with this condition will be created in the process of learning, in which students free expression and gives an opinion, so that students will participate actively in teaching and learning.

One type of approach in learning mathematics is a realistic approach. According to the [3] stated that learning mathematics with the realistic approach is an approach in which mathematics is seen as something human activities. The principal activities done in learning mathematics with the realistic approach are (1) using real-life contexts as a starting point for learning; (2) connecting to among strands, to other disciplines, and to meaningful problems in the real world; (3) using models as a bridge between abstract and real, that help students learn mathematics at different levels of abstractions; (4) using student's own production or strategy as a result of their doing mathematics; and (5) interacting as an essential for learning mathematics between teacher and students, students and students. By using a realistic approach, students doing troubleshooting informally (using its own language), but after some time the familiar with the processes of solving similar, they will use more formal language and end the process students will find an algorithm.

This research includes the type of research development that the research oriented on the development of a product development process are described

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carefully and products obtained have been evaluated. The product of this research is a mathematics instructional model based on RAKIR (Child Friendly, Innovative , Creative and Realistics). The development model of learning mathematics based on RAKIR is the main activity in this research. The development model used to develop this learning model refers to the model of development of public education from Tjeerd, [7]. It consists of four development phases: (1) the preliminary investigation phase conducted to get information about mathematics instructional model theories, learning theories and the analysis of the mathematics at Junior High School, (2) the design phase conducted to design the instructional model based on RAKIR, (3) the realization phase conducted to develop mathematics instructional model by following the design phase and (4) the revision, evaluation and test phase of the mathematics instructional model prototype being developed and was validated by an expert in learning mathematics and try out.

This research was conducted at the Junior High School Muhammadiyah Program Khusus Surakarta grade VII. The source of the data for the validity of the model is an expers competent in the fields of the development of the model and the source of the data for the practicality of the model are an expert in the field of development, researchers and teachers who carry out learning . The source of the data for the effectiveness of the model are students.

The data analysis techniques used in this research was to know the validity of the model are

(a) do the recapitulation statement validator and (b) determine the validity of the results of the compatibility with the criteria already defined. As for knowing the practicality of the model, namely (a) do the recapitulation statement validator and (b) determine the practicality with compatibility results with criteria that are already determined. And to know the effectiveness of the model at the end of the trial will be given the question form of student response about implementation models as well as the activity of students in learning.

3. Result and Discussion

The results of the development of mathematics instructional model based on RAKIR of every phase are as follows:

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1. The results of the preliminary investigation phase, i.e. the concept and theory of the curriculum of 2013, child friendly school (child-friendly), creative learning, innovative and realistic learning model and analysis of the material fractions grade VII about comparing fractions and operations of fractions.
2. The results of the design phase are : (1) design of mathematics instructional based on RAKIR with the phases of the syntaxes are as follows (a) explaining the learning objectives and motivating students, (b) giving the contextual problems that are familiar with students, (c) processing mathematics' abstraction (mathematics vertically), (d) formulating the solving strategies, (e) communicating the result of discussion and (f) giving the inferences of mathematics material; (2) the design of the learning environment or social system, i.e. the situation or atmosphere and norms that apply in the model of learning that will be developed , such as the role of the teacher and the student to do activities for learning to take place, (3) the principle of reaction, that is related to how teachers in the notice and treat students in the learning process, (4) supporting model based on RAKIR, i.e, materials/devices/media tools and learning that supports the implementation of the model and (5) the evaluation is to evaluate achievement of learning objectives pertaining to the mastery of the material with a learning model based on RAKIR.
3. The results of the phase of realization is a model of learning mathematics based on RAKIR prototype I .
4. The results of the test phase, evaluation, and revision are twofold, namely : (a) the results of the validation and (b) the results of the field trials. The results of the validation showed that learning model based on RAKIR that developed including the requirement is valid, because it meets the validity of the constructs and content. From the results of trials conducted in field indicates that:
   a. At the first trial, as seen from the practicability of learning models showed the ability of teachers in the management of learning including categories is not good. This indicated the implementation of learning syntax RAKIR not yet implemented properly, with a score of 2.75. While judging from the model effectiveness is measured from the activity and the response of students showed that 65% of students include the requirement of active in

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the learning process and responded positively towards the implementation of the learning model RAKIR. So it can be said that the model of learning mathematics based on RAKIR developed empirically is not practical and effective.

b. In the second trial, as seen from the practicability of learning models showed the ability of teachers in the management of learning including categories less well. This indicated the implementation of 2 stages in learning syntax RAKIR have not been executed and the score obtained in the learning management 3.0. So it can be said that the model of learning mathematics based on RAKIR developed empirically is not yet practical. While judging from the model effectiveness is measured from the activity and the response of students showed that 75% of students include the requirement of active in the learning process and responded positively towards the implementation of the learning model based on RAKIR. So it can be said that the learning model has not been effective.

c. On the third trial , as seen from the practicability of learning models showed the ability of teachers in the management of learning including categories either. This indicated the implementation of all stages of the learning RAKIR syntax was implemented by a score of 3.5o. While judging from the model effectiveness is measured from the activity and response indicates that 90 % of students include the requirement of active in the learning process and responded positively towards the implementation of the learning model based on RAKIR. So it can be said that the model of learning mathematics based on RAKIR developed empirically is already practical and effective.

Thus, after the last trial then it obtained a prototype of the final mathematics instructional model based on RAKIR that is valid, practicality and effective. Implementation of this model will be implemented in the second year, i.e. the year 2016. From the results of those trials indicate that the model of learning mathematics based on RAKIR can increase the ability of teachers in the management of teaching and students' activities in learning as well as responded positively towards the implementation of this model.

This condition as a result of the implementation of the learning that begins at the initial phase of mathematics instructional model based on RAKIR that is the teacher giving the contextual problems that are familiar with students. The teacher provides contextual problems. In this phase the teacher have to creativity and innovation in finding a contextual problem, so as to stimulate the creativity of the students. Students identify the problems individually. Based on their identification, students discuss the problems in their group in order to know the similarities and the differences of students' understanding of the problems. Afterwards, they have the same understanding regarding the problems. Students identify the problems individually. Based on their identification, students discuss the problems in their group in order to know the similarities and the differences of students' understanding of the problems. Afterwards, they should have the same understanding regarding the problems. The teacher encourages students to express their ideas. Each student in every group presents their ideas about the problems given and makes relations to other concepts or to realistic situations. Students in their group formulate the mathematics model based on their ideas. The model can be a bridge to connect the realistic problems and the abstract forms. This research in line with [1], [12] statement, that during the course of the study, an effort was continually made to encourage students to go troug the RME approach of simplifying the contextual problem by first represeting it in their own symbols and/or words and then futher solving and interpreting it from there.

4. Conclusion

The conclusions obtained in this study are:

1. A theory of development which is used to develop a model of learning mathematics based on RAKIR is modifying the development theory has been said by [7] which contains the phases (1) the preliminary investigation phase (2) the design phase , (3) the realization phase and (4) the revision, evaluation and test phase.
2. The learning management based on RAKIR by teacher was in " very good" category with score 3,50 ,

3. The prototype model had improved students' activities were in "very good" category and the students' responses toward the mathematics instructional model was included in "positive" category.
4. The instructional mathematics model based on RAKIR with the following syntaxes: (1) explaining the learning objectives and motivating students, (2) giving the contextual problems that are familiar with students, (3) processing mathematics' abstraction (mathematics vertically), (4) formulating the solving strategies, (5) communicating the result of discussion and (6) giving the inferences of mathematics material.

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Problem Solving Ability of Students to Solve Ordinary Differential Equations

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ABSTRACT

This study aims to describe the problem solving ability of students to solve ordinary differential equations (ODE). The research is a quantitative descriptive with twenty-eight students of fifth semester mathematics education as subjects who enrolled in ODE. The test method is used to determine the strategy of students to solve contextual problems of ODE. This study uses Polya framework as the basis for the analysis of the problem solving ability of students. The result shows that most of the students (approximately 67,9%) are able to write the information in the question but incomplete. No one able to write down the information of the problem correctly. Furthermore, almost 78,6% of students can not write the formula or model correcly in planning step. Nearly 10,7% students are able to plan the strategy to solve the problem properly and only a student perform correct steps to produce right solution. This results indicate that based on the Polya framework most of the students unable to solve the contextual problem of ODE correctly. In other words, it shows the poor ability of students to solve contextual ODE.

Keyword: contextual ODE, first order differential equation, problem solving ability

1. Introduction

Problem solving ability is the most important objective in learning mathematics. According to the Mathematical Association of America (MAA): "*every college graduate should be able to apply simple mathematical methods to the solution of real-world problems*". In addition, MAA also stated that "*college students should be expected to go beyond routine problem solving to handle situations of greater complexity and diversity, and to connect ideas and procedures more readily with other topics both within and outside mathematics*" [2]. This indicates that according to MAA, students is expected to solve the real problems or contextual (*real-world problem*) using the knowledge, concepts, ideas, and procedures that have been studied. Students should be able to solve problems outside mathematics using mathematical knowledge he had. Furthermore, NCTM (National Council of Teachers of Mathematics) in Principles and Standards for School Mathematics [3] states: "*A major goal of mathematics is to equip students with the knowledge and tools that enable them to formulate, approach, and solve problems beyond Reviews those that*

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they have studied ". This means that according to the NCTM the capability to use knowledge to solve math problems is the essence of learning mathematics.

ODE is a branch of mathematics which it studied not only mathematics students but also the students who majored in engineering, science, and even the social sciences such as economics and psychology. The applications of DE are in wide areas such as problems of direct current (engineering), rotation of objects around the other object (Physics), the population growth rate (Geography), the rate of growth of bacteria (Biology), the rate of chemical reactions (chemical), the rate of study (Psychology), as well as the rate of investment (economic). Thus the extent of problems that can be formulated using ODE.

The effective learning strategy of ODE is a challenge for teachers. Some researchers have attempted to develop an effective learning strategy ([1], [7]). In the learning of ODE, the students are required to connect mathematical concepts that have been studied in differential and integral calculus. The inability to connect the concepts in calculus will lead to weak mastery concept of ODE. Students will only be stuck to solve ODE procedurally [7].

This study aims to describe the problem solving ability of students of mathematics education who enrolled ODE courses in odd semester 2014. To analyze the problem solving ability of students used the Polya framework [4] which includes four steps, namely understanding the problem, devising a plan, carrying out a plan, and look back results obtained. At stage of understanding the problem, students must clearly understand what variables that exist in the problem, what data is needed, and what conditions are met or not met in problem. Second stage is devising of the plan that is the students have to be able to make connections among concepts in the problem. The ability to make connections will help them in planning the appropriate solution. In contrast, the weakness in understanding the connection will lead to errors in the plans drawn up to solve the problems. In the third stage, the students carry out a step-by-step plan that has been prepared in the second stage. Students should also concern with accuracy so that each step is done correctly. In the last stage, the solution has been generated look back to evaluate the concepts used, believes that the measures undertaken are correct, and minimize the chances of a mistake in the solution.

2. Method

This research used a quantitative descriptive approach. Twenty-eight students of fifth semester of mathematics education program are chosen as subjects. The data were collected through tests and documentations. Then the students answer were analyzed using four steps Polya strategies namely understanding the problem, devising a plan, carrying out a plan, and look back the results were obtained. In each step was classified into three criterias to describe problem solving ability. Furthermore, the problem solving ability of the students is also presented in percentage.

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3. Result and Discuss

The application problem of the differential equation is given to find out problem-solving ability of students with regard to the strategies used by the student to solve a problem. Their strategies could be indicate the problem solving ability of students. The contextual problem of ODE is as follow.

Problem:
A bowl of hot soup that has just been cooked have temperatures of 100^o C and placed in a room temperature of 0^o C. After 30 minutes the temperature of the soup decrease to 20^o C. When the velocity of temperature change (cool down velocity) is proportional with difference between soup and room temperature, determine temperature of soup after 60

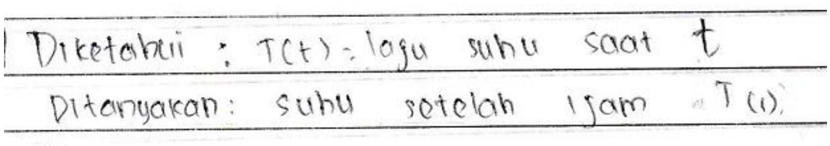
To analyze how students solve the problem above will be seen from a four-step problem-solving strategies of Polya. At each step was classified into three criterias which different in every step.

Step 1. Understanding the problem

The ability of understanding the problems can be classified into three criterias as follows:

- Able to write down the information of the question correctly
- Only capable of writing information partly
- Can not write any information on the question

Based on an analysis of student answers, find that no one able to write down the information of the problem correctly. Most of the students (approximately 67,9%) are able to write the information in question but incomplete. They forgot to write down information about proportionality between the velocity of temperature changes and the difference temperature between room and soup. They only focus on the value of temperature at a time. Finally, about 32,1% students do not write any information on the question. They tend to solve the problem directly without write first known information and asked of the problem. Moreover, all of their answer are incorrect. Here are some examples of how students write down the information of the given problem.


Diketahui : T(t) = laju suhu saat t
Ditanyakan : suhu setelah 1 jam = T(1)

1a

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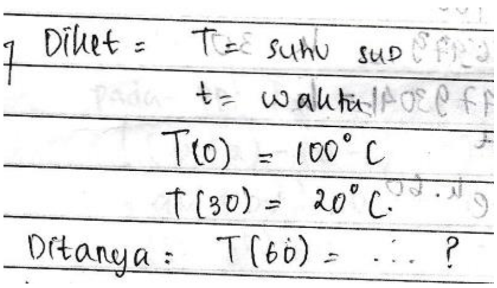
Diket = T = suhu sup
t = waktu
T(0) = 100° C
T(30) = 20° C.
Ditanya : T(60) = ... ?

1b

Figure 1. Incomplete information

Figure 1a shows that the students only provide general information about temperature change without inform the temperature of soup at $t = 0$ and $t = 30$. In Figure 1b, even though the information provided more complete than 1a, it also have not showed information that the velocity of temperature change is proportional with difference temperature between room and soup. The results of interviews with students shows that they feel have to write what is known and asked of the problem. But, they tend to write information relating to computation only. While other important information such as the proportionality between the velocity of change in temperature and the difference temperature between room and the soup is not concerned.

Step 2. Devising a plan

In devising a plan, the students have to be able to make connections among concepts in the problem where it can be presented in the form of formula or model. This step was classified into three criterias as follows.

- Write down the formula or model correctly
- Formula or model is written but incomplete
- Can not write the formula or writing incorrect formula

An analysis of the students answer obtained that almost 78,6% students can not write the formula or model correcly. They made an errors in writing the proportionality relationship between the velocity of temperature change and the difference of temperature between room and soup. It seems they do not understand the concept of proportionality. Only 10,7% students able to write down the formula or model correctly. The same percentage of students who appears to be able to write formulas or models though incomplete. Some examples of devising a plan according to the students answer are as follows.

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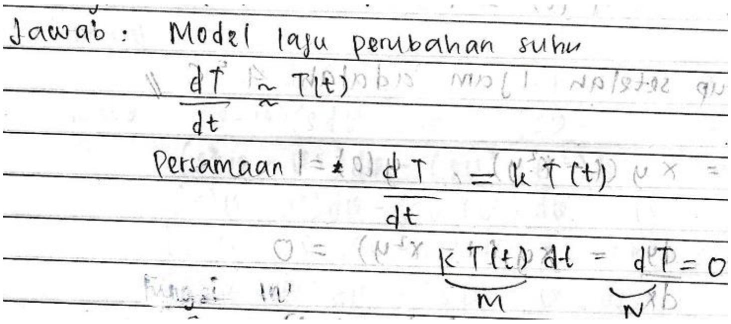
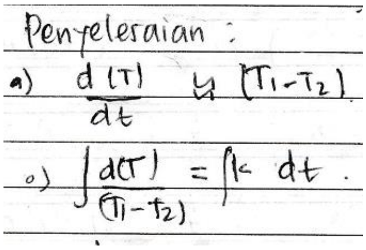
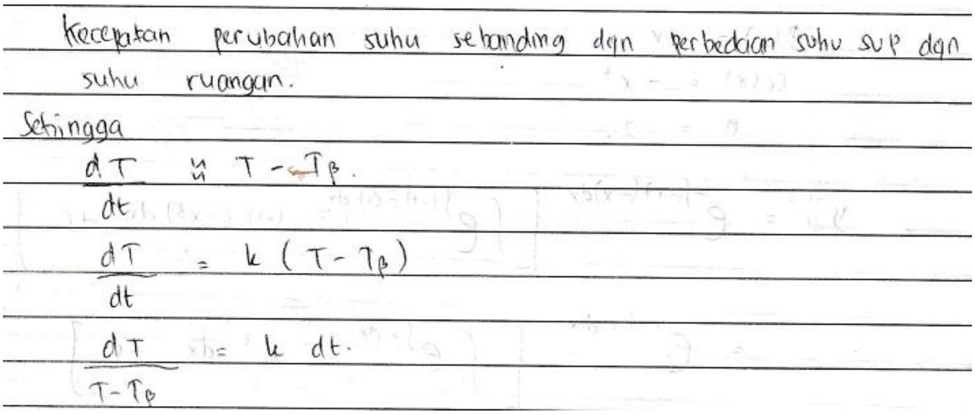


2a



2b



2c

Figur 2. Students answer for devising a plan

Figure 2a shows the incorrect formula or model written of the students in solving the problems. They can not use the information appropriately that the velocity of temperature change is proportional to the difference temperature between room and soup. In Figure 2b, the students have been able to write relationship among the velocity of temperature change and difference temperature between room and soup properly. But the next step occurs stepping step solution. First, they should be formulate the equation by multiplying a constant *k*. Then it can be solved by integrating the equation obtained. The correct planning strategy to solve the problem is presented in figure 2c.

Step 3. Carrying out a plan

In the third step, the students carry out a step-by-step plan that has been prepared in the second step. Students should concern with accuracy so that each step is done correctly. This step was classified into three criterias as follows.

- Correct steps of the solution and produces the correct answer
- Correct steps but make the wrong answer, or incorrect steps of the solution
- Not able to perform the steps of solution

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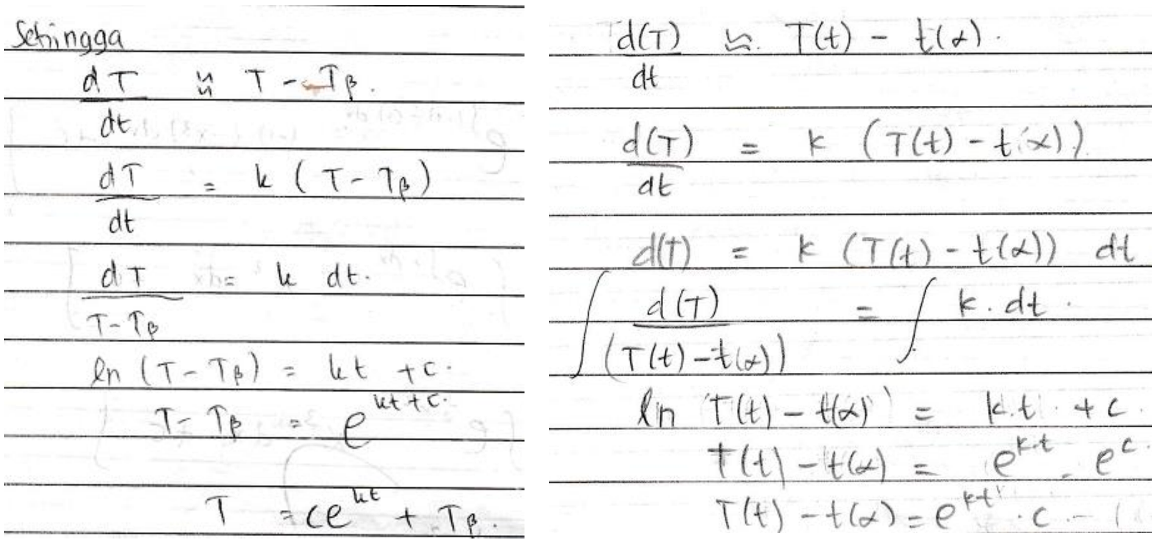
According to the results of the second step, only three (approximately 10,7%) students able to write down the formula or model correctly. Besides that there are also three students who are able to write the formula or model although incomplete. In this steps, we analyze six documents of students answer who able to write the formula or model both correctly and incomplete. The result shows that only a students who was able to carry out the formula correctly and produces the correct answer. Five other students perform incorrect steps to solve the problem and another students who can not write the plan in the second steps naturally can not perform the steps of solution. Examples of the students answer in carrying out a plan are presented as follows.



3a **3b**

Figure 3. Students answer of carrying out a plan

Figur 3a shows the incorrect steps perform by students. They carry out the erros at the third row to fourth in their answer. They do not use integration to obtain the result in the fourth row. Conversely, the errors made by students in the figure 3a does not occur in students whose results are shown in Figure 3b. He was able to write down the steps to solve the problems systematically. Furthermore, the student is able to solve the problems given appropriately.

The last step of Polya strategies is looking back where the solution has been generated look back to evaluate the concepts used, believes that the measures undertaken are correct, and minimize the chances of a mistake in the solution. Unfortunately, it is very difficult to understand whether the student has looked back or not based on the result of students answer. Therefore no information can be analyzed on the last step of Polya strategies.

These study results indicate that in order to solve problems correctly, firstly the students must understand the problem stated by writing the information, finding the necessary information, and write down what is asked in the problem. Secondly, must be able to connect the information obtained in the first step by other concepts that they have learned to arrange them into proper formula or model. Thirdly, students must be carry out the plan which have been prepared accurately to minimize the errors occur. The last step necessary to ensure that all steps we are doing is correct.

This study results confirm the other research such as Rowland and Jovanowski [5]. They examined the difficulties the student in solving the first order differential equation. Diagnostic quizzes, exams and interviews are used to collect data. The research showed that most students do not understand the functions and equations that must be determined, and most of them do not realize what it requires to solve the problem of differential equations. In other study, Sanchez [6] have observed certain errors that students make when solving an ordinary differential equation. He concluded that the student does not recognize the type of ordinary differential equation correctly and it cause mistakes in solving process. Thus the failure to understand the problem or identify any information in the problem cause the failure to solve the problems.

4. Conclusion

The problem solving ability of the students to solve ODE can be seen by using Polya framework which consists of four steps namely understanding the problem, devising a plan, carrying out a plan, and looking back the solution. The result of this study shows that most of the students (approximately 67,9%) are able to write the information in the problem but incomplete. Apparently they did not pay attention to some other important information. Almost 78,6% students can not write the formula or model correcly which effect only a student who was able to carry out the formula correctly and produces the correct answer. In other words, they have poor ability in problem solving of contextual problem ODE.

These results provide an overview of problem solving ability of students in solving contextual problem ODE. However, this study is limited to the analysis of the documents on the student's answers in solving the problems given. Further analyzes through interview will allow obtaining more complete information about problem solving ability of students.

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Developing Students' Mathematical Communication Through Realistic Mathematics Learning

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ABSTRACT

The purpose of this study is to determine the achievement and improvement of students' mathematical communication ability in Bengkulu City Junior High School through realistic mathematics learning. The method used is a quasi-experimental non-equivalent control group design. The results of this study as follows. The realistic mathematics learning is more effective in the achievement and improvement of students' mathematical communication ability than conventional learning. The magnitude of the achievement of students' mathematical communication ability in realistic mathematics learning is 63,96 and conventional learning is 47,46. Meanwhile, the magnitude of the improvement of students' mathematical communication ability is realistic mathematics learning is 0,51 and conventional learning is 0,24 .

Keyword: mathematical communication ability, realistic mathematics learning.

1. Introduction

The mathematical communication ability in secondary schools receive less attention from teachers. Teachers tend to emphasize on numeracy, problem solving, and reasoning. So that weak students' mathematical communication ability. Students are less able to communicate mathematical ideas clearly and correctly, both orally and in writing. Rohaeti [1] research results, Wihatma [2], Purniati [3] concluded that the ability of mathematical communications junior secondary students is low.

The mathematical communication skills is one of the standards in the process of learning mathematics in school. Van De Wall, Karp, Jennifer, and Williams [4] suggested 5 standards in mathematics learning process are: 1. problem solving, 2. reasoning and proof, 3. communication, 4. connection, and 5. representation. The standard process communication capabilities provide restrictions on communication aspects that are taught in school. The mathematical communication skills is the ability of students to communicate mathematical ideas to others.

Baroody [5] explained that mathematics learning should help students to communicate mathematical ideas through five aspects: representing, listening, reading, discussing, and writing. Baroody [5] also explained that the importance of communication in the mathematics learning, because mathematics as language and mathematics learning as a social activity. As a language, mathematics is used in conveying the idea by using symbols and understanding which has a single meaning. Mathematics is used in schools in mathematics produces a wide range of activities of students and teachers in discussing mathematics. In addition to classroom activities (schools), mathematics is also used by the communities in their social activities. As in trading activity, agriculture, mining and others.

To overcome the disadvantages of students mathematical communication abilty is necessary to change the learning, from conventional learning to non-conventional learning. Firdaus [6] explained that the non-conventional learning increases students' mathematical communication ability. One of the non-conventional learning is realistic mathematics learning (RML). Considers that the realistic mathematics learning of mathematics as a human activity. As an activity, mathematics as a tool that allows the interaction between humans.

Some research indicates that the RML is able to enhance the students' learning activities. Both physical and mental activity. Because RML includes a variety of physical and mental activity. Such discussion, reflection, and discovery of concepts and algorithms. Khasanah [7] found that the Indonesian Realistic Mathematics Education (IRME) can increase student activity in learning around the flat and wide awake. Likewise, research Al Muhari [8] produce that IRME can improve students' creativity.

1.1. The Researc Problem

a. Is there an increase in students' mathematical communication ability is taught through realistic mathematics learning?
b. Is student achievement mathematical communication ability is taught through realistic mathematics learning higher than conventional learning?

2. Theory

2.1 The Mathematical Communication Ability

Literally, the communication means of sending and receiving news or messages between two or more people (Surayin [9]). This means, in communication contains three components, namely the content of the message/news, the process of sending/receiving messages, and the people who send/receive the message. In the context of mathematics learning, the contents of the message/news in the communication, namely mathematics. Suriasumantri [10] suggested that mathematics is a language that symbolizes the meaning of a series of statements that we want to convey. Shaped mathematical symbols that have specific meanings. These symbols may include facts, concepts, principles and algorithms. Examples fact is bilngan π, variable as a concept, the pythagorean theorem as a principle, and steps to resolve the arithmetic operation two numbers is an algorithm.

By the term, Greenes and Schulman [11] suggested a mathematical understanding of communication as an ability to: a. stating mathematical ideas through speech, writing, demonstration, and describe it visually, b. understand, interpret, and assess the ideas presented in writing, verbal, or visual, c. construct, interpret, and connect the various representations of ideas and relationships. Based on this understanding, mathematical communication more detail. The message content/news in the form of mathematical ideas. The process of sending/receiving the message content/news in the form of written, oral, representations. National Council of Teachers of Mathematics [12] describes the shape of the communication process mathematically as: create illustrations and interpretations, talk or discuss, listen or hear, write, and read. Form of activity the process is declared, understand, interpret, and connecting the message content (mathematics).

The mathematical communication abilty is an ability to communicate mathematically. Hulukati [13], describes the communication of mathematics as a student's ability to express, interpret, evaluate ideas and mathematical notation through written, oral, and demonstrate it verbally. From understanding the mathematical communications capabilities have characteristics. Sumarmo [14], describes the characteristics of mathematical communication abilty as follows:

a. Make the connection real objects, drawings, and diagrams into a mathematical idea.
b. Explaining ideas, situations and mathematical relationships orally and in writing with real objects, images, graphics, and images.
c. Declare a daily occurrence in the language or mathematical symbols.
d. Listening, discussing, and writing about mathematics, reading with understanding a mathematical representation.
e. Make a conjecture, make the argument, a definition and generalization.
f. Explain and make inquiries about mathematics.

Van De Wall, Karp, Jennifer, and William [4] sugested an indicator of the ability of mathematical communications student at the school as follows:

a. Organinize student mathematical thinking through communication.
b. Communicating student mathematical thinking coherently and clearly to friends teachers.
c. Analyze and evaluate the mathematical thinking.
d. Using the language of mathematical ideas precisely.

These indicators show the communication skills to a student or a teacher. While the National Council of Teachers of Mathematics [12] describes indicators mathematicak communication ability as follows:

a. Revealed the mathematical ideas in writing or verbally.
b. A definition and make generalitations.
c. Presenting mathematics with understanding.
d. Explaining mathematical questions.
e. Appreciate the power and beauty of mathematics.

According Hendriana [15], about the communication activities related to activities reflecting mathematical cognitive processes, describing the procedure, described metacognition, and communicate with others about mathematics. Activities in mathematical communication is recorded mathematically and represent something with the symbol. While communicates with mathematics as follows: mathematics as a

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means of solving the problem, look for alternative solutions, interpret arguments, and use mathematical problem solving.

2.2. The Realistic Mathematics Learning (Rml)

Haji and Abdullah [16] suggested learning realistic mathematics as a systematic pattern in designing mathematical learning effective to achieve the goal of learning mathematics by relying on the creativity of the students in doing doing mathematics who view mathematics as a human activity through problem-solving activities contextual, formulate models , linking various topics, interact with a variety of sources, utilizing a variety of his own potential, discuss, reflect, take advantage of the phenomenon of education, explore, and finally found the (invention) concepts (principles) and mathematical algorithms. From that sense, has the characteristics of realistic mathematics learning. Treffers [17] describes five characteristics of realistic mathematics learning, namely: 1. The use of context, 2. The use of models, 3. The use of students', 4. The interactive character of teaching process, and 5. The intertwinement of varios learning strands.

3. Methodology

3.1. Sample and Population

The sample is students of class VIIa SMPN 24 Bengkulu City consisting of 19 students. The sample comes from a population of students of class VII SMPN 24 Bengkulu City consisting of 85 students (Haji and Abdullah [16]).

3.2. Instrument Validity and Realibility

The mathematical communication ability test consists of 3 items. The results of the analysis of mathematical communication ability tests using software Anates as follows (Haji and Abdullah [16]): mean = 60.79, standard deviation = 22.75, XY correlation = 0.71, reliability test = 0.83, item test = 3, and the number of subjects = 19.

3.3. Research Design

This study use a quasi-experimental design by the non-equivalent control group (Cohen, Manion and Morrison [18]):

Experimental	O1	X	O2
Control	O3		O4

X is a treatment as realistic mathematics learnining.

3.4. Data Analysis

Analysis of the achievement of mathematical communication skills using Mann Whitney test, if the data is not normally distributed. When the normal distribution of data using t-test. Where as the improvement of communication mathematical ability analysis using test N-Gein.

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4. Results and Discussion

5.1 The Increasing Students Mathematical Communication Abilty Through Realistic Mathematics Learning

Hypothesis test results showed that there is an increase students mathematical communication abilty taught through realistic mathematics learning. As shown in Table 1. Increasing students mathematical communication abilty taught through the learning of mathematics realistically is 0.51. While increasing communication skills of students who are taught mathematics through conventional learning of 0.24. Increasing students mathematical communication ability greater than the students who are taught through conventional learning.

Tabel 1. Test Results of t–test

Data Group	*N*	Average	Average Difference	*t*	*df*	*Sig.* (2-way)	H_0
N-*gain* Experiment Group	27	0,51	0,27	3,396	49	0,001	Rejected
N-*gain* Control Group	24	0,24					

Source: Haji and Abdullah [16]

The increasing students mathematical communication is taught through realistic mathematics learning happen, because learning realistic mathematics to motivate students to conduct discussions on their friends and teachers in solving contextual problems and in finding a concept in mathematics. As in solving the following problems, known integers 786 and -867. Where larger numbers? Why? As many as 95% of students who are taught through realistic mathematics learning the correct answer to these problems. The results of their discussions as follows:

Student A : 786 is greater than -867, as a positive number greater than negative numbers.

Student B : Try to point out that the positive numbers greater than negative numbers.

Student A : Well, I would point out that the number 786 is greater than -867, using the image of the number line as follows:

-867 0 786

Student B : Why -867 numbers located on the left number 786 and between the two of these numbers are zeros?

Student A : Because the number -867 is a negative number, while the number 786 is a positive number. Numbers 0 outweigh the negative numbers and smaller than positive numbers.

Student B : Oh, then I agree.

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A total of 73%, the students taught by conventional teaching experience difficulty in resolving the matter. A total of 62%, the student answers incorrectly. They say -867 is greater than 786. Since the number 8 at -867 is greater than the number 7 at 786. The discussion activity in less developed conventional learning. Generally, each student solved the problems.

4.2 The Achievement Students Mathematical Communication Ability Through Realistic Mathematics Learning

Mann Whitney test results showed that accepting the hypothesis that there are differences in the achievement of mathematical communication skills of students taught using realistic mathematics learning with conventional learning. Achievement of mathematical communication skills of students who are taught through realistic mathematics learning is greater than students taught through conventional learning. Mathematical communication skills of students who are taught through realistic mathematics learning at 63.96, while students taught through conventional learning at 47.46. This is shown in Table 2 below.

Table 2. Results of *Mann-Whitney-Test*

Data Group	Evarage	*U Mann Whitney*	*Z*	*Sig.* (2-way)	H_0
Pretest of Experiments Group	32,33	221,500	-1,950	0,051	Received
Pretest of Control Group	32,00				
Postets of Experiments Group	63,96	149,500	-3,306	0,001	Rejected
Postest of Control Group	47,46				

Source: Haji and Abdullah [16]

The students mathematical communication ability who are taught through realistic mathematics learning faster than students taught through conventional learning. Due to the realistic mathematics learning, student reflection and discovery activities. Through reflection, students have the opportunity to correct the error or lack of inaccuracy in resolving a problem. The reflection of student activities conducted by reviewing the things he had done. Students communicate with itself. Similarly, the discovery activity. Students communicate with a friend or teacher. As in finding the circumference of the triangle. Students use a rope to rounded triangular-shaped object. So that the students concluded that the circumference of a triangle is the sum of the three sides. Through this understanding, as much as 77% of students can answer correctly the following question. The circumference of a long triangle sides (a + 6) cm, 28 cm and 12 cm is 180 cm. What is value of a? Answer that question is a = 134. One way to answer that question as follows:

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The three sides of the triangle drawn in a line along the length.

(a + 6) cm 28 cm 12 cm

a + 6 + 28 + 12 = 180 cm

6 + 28 + 12 = 46

a = 180 – 46 = 134

Image increasing and achievement of mathematical communication ability who are taught through realistic mathematics learning as well as through conventional learning is shown in Figure 1 below.

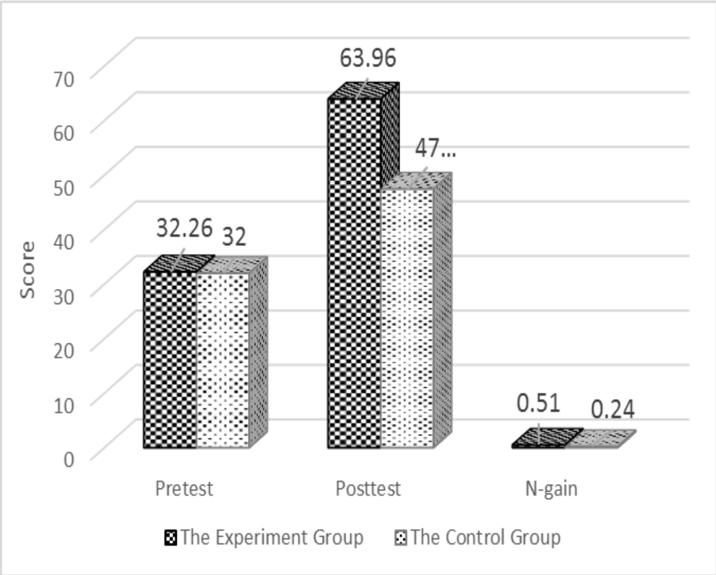

Figure 1. Increasing and Achievement Students Mathematical Communication Ability

Source: Haji and Abdullah [16]

5. Conclusion and Suggestion

The results of this study are:

a. There is an increase of students mathematical communication ability are taught through Realistic Mathematics Learning at 0.51.
b. The achievement of students mathematical communication ability taught through Realistic Mathematics Learning higher than students taught through Conventional Learning. Realistic Mathematics Learning achievement through at 63.96, while the achievement by conventional learning at 47.46.

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Dwi's Concept Understanding Concerning Operation on Integer

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ABSTRACT

This was qualitative research. The research subject was Dwi. She was Grade V.B student of SDN InpresTamalanrea I Makassar. The research objectives were to reveal her type and level of understanding on mathematics concept in the topic of operation on integer. In investigating her understanding, she was provided with a test, and then undertaken an unstructured interview. The research results show that: (1) her score is 67 of an ideal score 100. Thus, she is in the moderate level (45 < the obtained score < 77), and (2) based on the interview outcome, it is concluded that her understanding is instrumental understanding.

Keywords: Concept Understanding, Operation on Integer.

1. Introduction

Mathematics consists of concepts organized hierarchically, structurally, logically and systematically beginning from simplest concept to complexest one. Students sometimes perceive difficult to understand mathematics concept and conveymathematics ideas, either orally or in writing. This is caused by some reasons.One of them is that the study object of mathematics is abstract. In addition, authors also frequently find students who are not able toimplement mathematics concept that they have learned. Jenningand Dunne Caray [1] states that most students feel difficultyin applyingmathematics in real life situation.

In 29 March 2015, it was conducteda research by providing for 'Dwi'aquestion in terms of operation on integer. Her work was: 8 x 2 – 2 + 4 : 2 x 3 = 16 – 6 : 6 = 15. This indicated that her mathematics concept still needed to be enhanced. Misconception in mathematicswill result in the lack of her wholly mastery.Moreoverthat in the basic concept,it will lead her to the difficulty to masterya higher concept.

This was also made firm with a study that Priatnaundertook Fauziah[3] concerning the ability for mathematical reasoning and understanding ofState junior High Schools in Bandung. The results are a considerable part of students acquired only the algorithm skill, without understanding its concept. Whereas we see Jerome Bruner Hildayanti [2] stating that, "Learning mathematicsmeans learning about mathematics concepts and structures, as well attempting to find relations between the concepts and the structures themselves". Therefore, students are then demanded to be more active during instructional process in order that they could grasp mathematics concept and structure covered in a learning material.As a consequence, they would understand content and mastery it well. Learning mathematics must also be administered step by step continuously due to the mathematics contentthat students learn is interconnected one another. Higher the level of students' education, higher the difficulty level of mathematics content.

Concept is a set of information that can be used to group or classify a set of objects Soedjadi [4] It is frequently named as building block for thinking that is basis for higher mental process to formulate principle and generalization. For solving problem, students must know relevant rules.These rules are based on the concepts acquired. Learning concept constitutes the main result of education.

A student is said to be having understood concept, if he orshe has been able to recognizing and abstractingsimilar attributesconstituting typical thingof the concept learned.In addition, he or she could generalize that concept. It means that if a child understands aconcept, he or she is thenable to identify and generalizean object in a variety of situations that are not used in learning situation. In other words, students can carry out various new problems requiring higher order thinking ability.

The Directorate General of Primary and Secondary Education of Indonesia No.506/C/PP/2004 Hildayanti [2] states indicatorsshowing concept understanding, they are:

a. Restating aconcept or definition. The first thing that a student should do to be considered ashaving ability in understanding concept that isrestating a concept or definitionusing his or her own words;
b. Classifying objectsaccording to certain attributes suitable with the concepts;
c. Providing examples and non-examples;
d. Presenting concept in a variety of mathematical representation forms;
e. Evolving necessary condition or sufficient condition of a concept; and
f. Applying conceptor algorithm into problem solving.

Understanding mathematics conceptsneed to considerpreviousconcepts. This is because concepts in mathematicsconstitutea series of causality. A concept is arranged on the basis of prior conceptsand will be basis for the subsequent ones.Therefore, understanding of a concept is an extremely significant for students to have in a learning.

Building on the aforementioned outline, it is then formulated a problem as, "what is her type and level of understanding on mathematics concept in the topic of operation on integer?"Thus, this study aims to reveal her type and level of understanding on mathematics concept in the topic of operation on integer.

2. Method

This was qualitative research aimed at analyzing Dwi's concept understanding to operation on integer. This research was conducted at Dwi's house for two days, namely 18–19April 2015.The research subject was'Dwi',a grade V.B of SDN Inpres Tamalanrea I. Dwi was a fictitious name for her.She was chosen as the research subject by considering: (1) her house was relatively easy for the researchers to reach out, and (2) her readiness to participate actively to support researchersduring data collection.

The research instruments are the researchers themselves. In this term, researchers are planner, data collector, analyzer, data interpreter, and at last as the reporterof the research results. The researchers also use supporting instruments, that istestand interview protocol. The testis suchquestions about operation on integer. The results are then used as the reference for categorizing students' mathematics understanding. For interview, researchers use unstructured interview.The stages for collecting data within this research were as the following.

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a. Test was provided for the research subject;
b. Checking students' test result using assessment rubric.
c. Based on the test result, researchers then categorizing her into one of threelevels, low, moderate or high level of understanding.

Table 1 Rubric for Assessing Dwi's Test Result

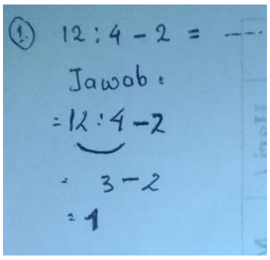
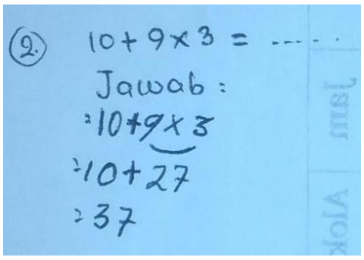
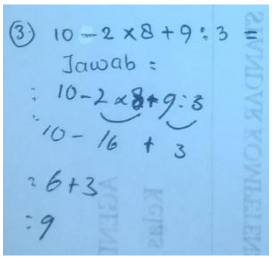
Rubric	Score
Solution is appropriate and answer is correct	3
Solution is lack appropriate and answer is correct	2
Solution is inappropriate and answer is incorrect	1
No answer	0

d. Subsequently, it was conducted interview to learn or trace the reasons of the research subject in answering the questions given; and
e. The subject's understanding was then studied through her interpretation or representationwhen answeringinterviewer's questions.

3. Results and Discussion

Data was obtained through the subject's answers to the test provided and the interview resultsfordiggingin Dwi's concept understanding ofoperation on integer.Her mathematics ability score was 67 of an ideal score 100. It was categorized intomoderate level. Within this research, the researchers usedthree categories, that islow ($x \leq 45$), moderate ($45 < x \leq 77$), andhigh ($77 < x \leq 100$), where x is the obtained score.

The questions provided for Dwi along with her answers are as follows:

		
This answer is correct, so is scored 3	This answer is correct, but scored 2.The appropriate one should be10 + (9 x 3)=10+27=37. This would avoid her to do error such inquestion No.3, which is,doing it from back to front.	This answer is incorrect, so is scored 1. The correct one is 10 – (2 x 8) + (9 : 3) = 10 – 16 + 3 = -3

Data obtained through test are analyzed quantitatively, and it is supported with the qualitative data obtained from interview. The authors then usecodingfor interview data that is P referring to researcher,and SP referring tothe research subject.

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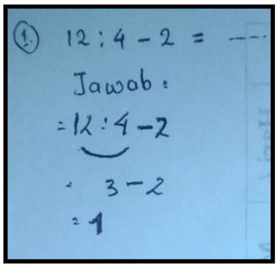
1. Question No.1

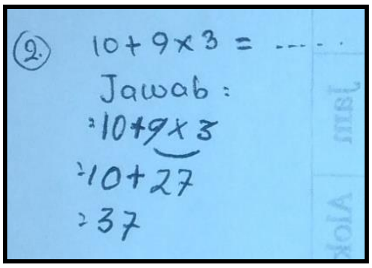


Interview:

P-01	Why is your answer like this, brother? 12 : 4 = 3 – 2 = 1
SP-01	Since I firstly divide (twelve) by four, and thensubtracted fromone, brother.
P-02	Why is the division done first?
SP-02	Because its value is greater than that one, brother.
P-03	What do you mean greater than or what is greater than like?
SP-03	I don't know, brother.(The expected answer is becauseit's *stronger than that one*).

The responsesabove show that 'Dwi' has been able to determine which operation should firstly be carried out. But 'Dwi' still perceives difficult in explaining the reason of choosing the division operation first to be done.

2. QuestionNo.2

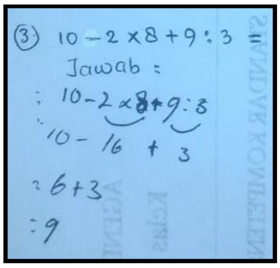


Interview:

P-04	Next, why is your answer like this?9 x 3 = 27 + 10 = 37
SP-04	Just the same as my answer in Question No.1, I multiply them first, and then add them.
P-05	Why is the multiplication should you do first?
SP-05	Just the same as in Question No.1, that is because it's greater than that one.
P-06	Is it because greater than or stronger than, brother?
SP-06	Yeah … that's all.
P-07	So, between multiplication and addition, which one is stronger?
SP-07	Multiplication, brother.

Based on the interview, ‘Dwi’ knows that operation ofmultiplication is stronger thanaddition. This is also strengthened by ‘Dwi’ within her last response.

3. QuestionNo.3



Interview:

P-08	Now, how do you know the answer to the Question No.3?
SP-08	I did it from back to front, brother.[*The expected answer is actually like this*:10 – (2x8) + (9:3) = 10 – 16 + 3 = -3]
P-09	How do you know that (from back to front)?
SP-09	I don’t know, brother.
P-10	I mean, why did you do it from back to front, brother?
SP-10	Since I do the stronger operations first, brother, like Question No.1 and No.2.
P-11	So, which ones are stronger?
SP-11	Multiplication and division, brother.
P-12	Then, why is that subtracted from 10 brother? [16 + 3 – 10 = 9]
SP-12	Because its sign is minus, brother. [*while appointing to*10 – 2 x 8 + 9 : 3 = …]

Here, Dwi is able to classifywhich stronger operationsofthe four basic operations on integer. But, from the solution to the question no.3, she did error in interpreting question with the sign (-10). This is because sheadministered it from back to front.

From these three outlines, it isinferred that Dwi’s understanding is instrumental. Instrumental understanding is indicated by someone’s abilityin using mathematical procedure for carrying out a problem by solely empowering formulas to be memorized, without knowing its rational reason.

4. Conclusion

The score of Dwi’stest result is in the moderate category (45 <the obtained score< 77). This is because the score obtained is 67 of an ideal score 100.Based on the interview results,it is obtained information that her concept understanding type is instrumental. Shehas an ability in using mathematical procedure to solve problem given by only dependingon the memorized formulas,without knowing the rational reason.Therefore, it is then suggested for any stakeholders to attempt to improve her understanding. This needs cooperation between teacherand parents in guiding Dwi. For instance, teacher at school may use any learning strategiesthat can make her learning meaningful for her. In addition, parents at home may also participate actively incontrollingtheir child’s learning.

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Analysis Mastery of Mathematics Teacher of Implementation Curriculum 2013 in the Junior School

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ABSTRACT

The curriculum used by teacher is unequal because of differences in teacher competence so the teacher mastery of implementation in the curriculum 2013 is very important. This study is a qualitative study that aimed to describe the teachers' mastery of the curriculum implementation in 2013 and gives an overview classroom atmosphere on the implementation of the curriculum 2013. Based on the analysis of the instruments that: 1). The ability of teachers to describe the ability of the topics of discussion on the subjects become interesting information and easily understood by students is enough; 2) The ability to identify the level and areas of difficulty students and ability to help him get out of these difficulties, is still lack; 3) The ability of teachers to the ability to evaluate students' progress is still lack;
Based on the results of this analysis are expected teacher can determine a strategy to determine more appropriate teaching methods and speed in providing information in the form of knowledge to the students. Teachers are able to identify the extent and area of difficulty students and help him get out of these difficulties and be able to evaluate students' progress. To related parties as LPMP, LPTK can assist teachers in addressing these shortcomings

Keywords: difficulty students, teacher mastery, teacher competence, the curriculum 2013

1. Introduction

The curriculum is a major factor in relation to teaching and learning activities. The curriculum plays an important and strategic because good understanding of the curriculum will support the creation of quality learning. Curriculum renewal in Indonesia so far has led to the Curriculum 2013.

a. Reason Entry Curriculum 2013 Can Be Viewed From Several Things:

A. Based on Future Challenges

- Globalization: WTO, ASEAN Community, APEC, CAFTA
- environmental issues
- advances in information technology
- convergence of science and technology
- economy -based knowledge
- revival of creative industries and cultural
- a shift in world economic power
- the influence and impact Technology and science
- Quality , investment and the transformation of the education sector

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- TIMSS and PISA results

B. Based Competence Future
- The ability to communicate
- The ability to think clearly and critically
- The ability to consider the moral aspects of a problem
- The ability to be an effective citizen
- The ability to try to understand and tolerant of different views
- The ability to live in a globalized society
- Have a broad interest regarding living
- Have a readiness to work
- Have the intelligence according to their talents / interests

C. Based on the negative phenomenon
- Fights students
- Drugs
- Corruption
- Plagiarism
- Cheating in Exams (contek, Kerpek)
- The volatility of society

D. Based on Public Perception
- Too focused on cognitive aspects
- The student load is too heavy
- Less -charged character

The curriculum as an instrument for improving the quality of education consists of three entities, namely the purpose, method, and content. Increased competence of teachers and the provision of educational facilities and infrastructure will only provide meaning for students if directed at achieving educational goals formulated in the curriculum. The formulation of the National Education System is formulated on Graduates Competency Standards (SKL). On Government Regulation No. 19 Year 2005 on National Education Standards General Provisions Chapter SKL is defined as" the ability of graduate qualifications that include attitudes, knowledge, and skills" To ensure that SKL can be achieved then the teaching and learning activities is equipped with seven other standards that content standards, standardized processes, standards educators and education personnel, facilities and infrastructure standards, management standards, financing standards, assessment standards. The existence of these standards has been guaranteed by the Government Regulation No. 19 of 2005, Article 2.

The education systems in Indonesia were not yet fully meet the National Standard of Education (SNP), which has been set by the government. LPMP

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cooperation with relevant agencies to implement a series of activities provided guidance to schools SBSNP models. Implementation of quality assurance systems in school education should direct the school to meet the 8 SNP. SBSNP model school has been committed to conducting quality education by always doing continuous improvement based on Quality Standards set out gradually in various activities.

Curriculum 2013 as part of interventions to increase the quality of education certainly cannot be contrary to the legislation in force. Therefore, SKL becomes a reference when the curriculum is implemented in 2013, including seven national standard of education. Likewise, the Education Unit Level Curriculum (KTSP) to remain part of curriculum 2013. Unit educational curriculum still has the authority to develop their own curriculum in accordance with the conditions of the educational unit. In addition, the curriculum in 2013 remains a curriculum based-competency. However, as stated in Law No. 20 of 2003 on National Education System Article 38, the basic framework and structure of the curriculum of primary and secondary education set by the Government. Education units still have to refer to the basic framework and structure of the curriculum if they have to develop their own curriculum. Provision to refer to the basic framework and structure of the curriculum is part of quality assurance. Based on the tendency of unequal curriculum used by the educational’s unit. This trend occurs because of the difference in competence of the teacher, so there is adopted the educational unit of the curriculum or the education unit and an example of the Curriculum Center of Books, without making adjustments to the conditions of the educational unit where the teacher teaches.

The successful implementation of Curriculum 2013 is not only on the accuracy and assess the broader scope (comprehensiveness) formulation SKL and the basic framework, as well as the structure of the curriculum, but from the principal's leadership at unit level of education and leadership teacher at classroom level. School leadership has an important role in facilitating teachers in implementing the learning process in the classroom. While the leadership of the teachers at the classroom level was not be separated by success in the implementation of Curriculum 2013. The teacher is a leading actor in the implementation of Curriculum 2013, which deal with the students. Furthermore, this study will focus on the implementation of the curriculum in 2013 at unit level SMP/MTs for Mathematics. This focus was selected to provide an explanation the important role of teachers, among others, include:

1. The ability to describe the topics of discussion on the subjects become interesting information and easily understood by students. 2. The ability to identify the level and areas of difficulty students and ability to help him get out of these difficulties. 3. The ability to evaluate student’s progress. Based on the results of the evaluation of the teacher can determine a strategy to determine more appropriate teaching methods and speed in providing information in the form of knowledge to the students.

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2. Methodology

This study is a qualitative study that aimed to describe the teachers' mastery of the curriculum implementation in 2013 and gives an overview classroom atmosphere on the implementation of the curriculum 2013. The crucial role of teachers, include:

1. The ability to describe the topics of discussion on the subjects become interesting information and easily understood by students.
2. The ability to identify the level and areas of difficulty students and ability to help him get out of these difficulties.
3. The ability to evaluate student's progress.

Collecting data using an instrument with a choice of yes or no answer, which observes the implementation of learning with aspects observed:

1. Activities Introduction; include: Recap and motivation as well as the delivery of competence and action plans.
2. Core Activities; include: mastery of learning materials, the application of learning strategies that educate, application of scientific approaches, use of learning resources/media in learning, the implementation of authentic assessment, involving of students in the learning and use of language is right and proper in learning
3. Activities cover; include: cover learning.

This study conducted in November 2014, is intended for math teachers at targeted schools SBSNP in three districts namely Bangka, Bangka Regency of Central and West Bangka in Bangka Belitung Islands province.

3. Result And Discussion

Collecting data on the results of preliminary activities to the activities ereperception result 68% of teachers answered yes and to plan activities 71% of teachers answered yes. For the core activities result 81% for mastery learning materials, 81% for the application of learning strategies that educate, 82% for the application of scientific approaches, 58% for the use of learning resources/media in learning, 95% for the implementation of authentic assessment, 75% for engagement students in learning as well as 100% to use correct and appropriate language learning. The activities cover the results obtained to cover 65% of learning.

Analysis of the **ability of teachers to describe the ability of the topics of discussion on the subjects to be informed of interesting and easily understood by students** is **enough** it is seen from the ability of teachers to the preliminary activities, the core and the cover where teachers have linked learning materials are now with the experience of students or previous learning; mastery learning material has very good, teachers can tailor the material to the learning objectives, presents a discussion of the right to associate the material with other knowledge relevant to the development of science and technology and real life, teachers have good control of the class, fostering the active participation of students in asking questions and expressing their opinions, facilitate and present activities for students to observe;

collect and process information; communicate and create, engage students in the use of instructional media; teachers demonstrate skills in the use of learning resources which are varied and use of instructional media and produce interesting message, the teacher has been well responded positively to the participation of students; showed an open attitude towards the response of students; demonstrate interpersonal relationships conducive; foster active participation of students through the interaction of teachers, students, learning resources; and generate excitement or enthusiasm of students in learning, teachers have used language clearly and fluently spoken and written language is good and true.

Analysis of **the ability of teachers to the ability to identify the level and areas of difficulty and the ability of students to help him get out of these difficulties**, is **still lack** it looks on the ability of teachers lacking in delivering the benefits of learning materials, asking challenging questions to motivate and demonstrate something that is associated with learning materials, did not submit a plan of activities, for example, individual, group work, and make observations. Teachers have not been systematically delivered material easy to difficult, from the concrete to the abstract, yet able to carry out the study in accordance with the planned time allocation, teachers are less fishing students to ask what, why and how. Teachers are very lacking in facilitating and guiding students to summarize the subject matter.

Analysis of **the ability of teachers to the ability to evaluate students' progress** is **still lack** as seen from the ability of teachers lacking in the implementation of authentic assessment, even if the teacher is good in providing guidance scoring, evaluation knowledge and skills is sufficient to provide oral or written test to collect the work as a material portfolio, facilitating and guiding students to reflect on the process and the subject matter; and follow up with giving direction following activities and tasks enrichment, but has not yet conducted an assessment of attitude and not master the techniques and instruments to assess achievement of competence and authentic assessment.

4. Conclusion

Based on the analysis of the instruments granted to target SBSNP school math teacher at the junior high school level concluded that:

1. The ability of teachers to describe the ability of the topics of discussion on the subjects become interesting information and easily understood by students is enough;
2. The ability to identify the level and areas of difficulty students and ability to help him get out of these difficulties is still lack;
3. The ability of teachers to the ability to evaluate students' progress is still lack;

Based on the results of this analysis are expected teacher can determine a strategy to determine more appropriate teaching methods and speed in providing information in the form of knowledge to the students. Teachers are able to identify the extent and area of difficulty students and help him get out of these difficulties and be able to

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evaluate students' progress. To related parties as LPMP, LPTK can assist teachers in addressing these shortcomings

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Development of PCL Approach in Mathematics Learning Integrated with Character Education at Junior High Schools in Gorontalo Province

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ABSTRACT

Character education is an integral part that is very important to be paid attention in education and learning process particularly in mathematics learning. Character education in learning activities was conducted by using integrated approach in a subject. The integration of education character in mathematics learning can be done through active approach. This approach is chosen because it is to build the character, the students are the ones who should have active role in learning process. One of active learning approaches was Problem Centered Learning (PCL). This research aimed at (1) investigating comprehensively the theories that are used to develop the PCL approach which is integrated with character education by following the stages of research and development model; (2) investigating comprehensively whether or not the developed learning tools is appropriate with PCL approach which is integrated with education character. This research was conducted through learning tools development stages. The process of learning tools development was using 4-D model modification according to Thiagarajan, Semmel & Semmel that consisted of define and design, develop, and disseminate stages. The final result of research gained that the knowledge that was developed through PCL based learning tools had contribution to the emergence of social interaction and negotiation.

Keyword: Character Education, Mathematics Learning, PCL Approach

1. Introduction

Character education is an integral part that is equally important to be paid attention in the teaching and learning process especially mathematics learning. Character education in teaching and learning activity within the class, implemented using the integrated approach within the learning thus, is expected to have instructional effects and nurturing effects for the students' characters development.
Each educational unit can determine which character values that would be emphasized that suits the school's character, the characters of the region, or the characteristics of the subject.

Integration of character education within the mathematics learning activity can be done by designing the learning activities using active learning approach explicitly and implicitly to develop certain character values. This approach is chosen due to in

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building character, students should be more actively participating during the learning process.

In correlation with the characteristics of 2013 curriculum, it is the author personal believes that what is meant by active learning approach is learning approach which is designed to: (1) the learning process is centered on students; (2) the learning process is interactive (interactive teacher- learners-environment-sources/other medias); (3) learning process based on multimedia tools; (4) learning process in the form of group learning (teamwork based); (5) learning process that emphasizes and nurtures the more critical and critical thinking of the students; and (6) the learning process that have an impact on the development of good characters values on students, [1]

Referring to the criteria above, in this research, one of the active learning approach is Problem Centered Learning (PCL) approach. PCL is one learning model that requires students' mental creativity to understand a concept through a situation and problem presented in the beginning of a lesson. This model is designed with the objective of students to be able to develop their own concept of understanding, involving the high thinking activity to create higher level of independence and self-confidence. According to Wheatley [2] this PCL approach is designed to give students to be more actively involved in the learning process by encouraging them to: (a) invent their own ways in solving some problems; (b) brainstorming on the level beyond arguments on what is wrong or right; (c) creative thinking beyond calculation written calculation.

PCL is divided into three components, namely, task, group, and sharing [3]. This approach is started by preparing the class through assigning some problem solving tasks to the students to accomplish those tasks. This activity is then followed by dividing students into small working groups and encourages them to collaborate. Upon the completion of discussion in each group, the last activity is the class discussion. In this session of the discussion, each group presents the materials that they have discussed within their group. It is expected that through this classroom discussion the sharing would happen thus, it would produce a solution to the problem at hand.

Based on the topic/content/materials aspect of mathematics subject in junior high schools or from the active learning approach discussed above, there are several characters values identified explicitly and implicitly that will be able to be constructed, developed and shaped in each mathematics learning session. Those values are vigilance, logical thinking, critical, self-confident, tolerant, responsible, and communicative, open mindedness, collaboration to learn from each other, take and give, and respect each other, optimism, ability to make quick and appropriate decision and constant self-reflection [4], [5], [6].

Therefore, it is clear that mathematics learning can facilitate development and nurture of character values, hence, can contribute in the nation's character building.

Implementation strategy that the teacher can apply is through identifying the character values that would be developed based on the topics within the mathematics subject, selecting the appropriate learning approach to develop and shape those values, create role model in implementing those values and evaluate it.
To implement that strategy in mathematics learning, conceptual and empirical study through a research to develop a learning approach using the PCL approach integrated with character education is needed. The developed learning approach is to optimize the students' roles as subject of the learning and to optimize the learning process in improving the students' ability and the establishment of students' character as part of the instructional and nurturing effects.

This research is aimed at designing and developing the PCL approach integrated with character education, through (1) a comprehensive study of theories used to develop the PCL approach integrated with character education by following the stage of research and development model; (2) comprehensive study of whether the developed learning tools is appropriate with the PCL approach integrated with the character education.

2. Research Method

This research is a research to develop learning tools such as, Lesson Plan, Learning Media, and Learners' Activity Sheets, and Learning Achievement Test.
The development model that would be used in this research is the research and development model of Four D model [7]. 4D model is chosen because it is more systematic and appropriate for developing learning tools, however, in this research, the researcher has modified the 4D model. This model is simplified into three stages, called define, design, and develop. The disseminate stage cannot be implemented because the objective of this research is to develop the learning tools to produce better learning tools emerge from the development stage.

The instructional development instruments used in this research are validation sheet, teacher's ability sheet and students' activity sheet, and learning achievement test. The data analysis for this research is the analysis of the validation of learning tools/instruction, analysis of teacher's ability in managing the classroom, and analysis on the activity of learners during learning process.

3. Research Result

3.1 Description on the Defining Stage

3.1.1 Initial-Final Analysis

The result of the observation and interview gathered the information that the learning activities were using conventional pattern in which learning are more dominated by the teachers. The learning activities were conducted where teacher describes the mathematics concept, gives examples and asks students to do the

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exercises. In learning activities, students tend to be passive and were not chance to construct their own knowledge independently.

Based on the interview with the mathematics teachers it was also revealed that the junior high school students' mastery of the mathematics topic is very low. Based on the cognitive learning theory, students have to be actively involved in learning to construct their knowledge for them to better memorize the topic. One of the better learning alternatives to encourage the students to participate actively in learning process is through PCL approach.

Implementation of PCL approach in learning needs appropriate learning tools. Therefore, the learning tools currently used by the schools are not appropriate for implementing this alternative learning approach; thus, a set of appropriate learning tools is needed to be developed to support the learning implementation.

3.1.2 Students' Analysis

In this analysis, several things as follows were revealed:

1. The socio cultural backgrounds of the students are diverse. The parents jobs are also various from teachers, civil servants, farmers, labor, merchant, business etc.
2. The junior high schools students' age is ranging from 11-15 years old. According to Piaget, their cognitive development is in the formal operation stage.
3. Based on the knowledge background of the students, the sub-topic of Comparison and Chance learnt by the students in grade VIII is not a new concept at all, because this sub-topic was once taught when they were still in elementary school. In addition, those topics are often found in their daily lives.

3.1.3 Materials Analysis

The materials analysis is aimed at identifying the parts of main themes that would be learnt by the students in Comparison and Change topic in grade VIII of junior high school that refers to the 2013 curriculum.

3.1.4 Task Analysis

Task analysis consists of general assignments and specific assignments. The general assignments refer to the basic competencies and core competencies in 2013 Curriculum. Meanwhile, specific assignments refer to modified indicators of learning achievement.

3.1.5 Specification of Learning Objectives

Specification of learning objectives are conducted by defining the basic competencies in learning achievement indicators specifically, based on the material analysis and assignment analysis.

3.2 Description of the Designing Stage

3.2.1 Media Selection Result

Learning media is needed in the implementation of learning using the PCL approach that has been adjusted with the material analysis, assignment analysis, learning objectives specification, and available facilities in the school. Based on those analyses, the media needed in PCL approach for those materials are white board, board markers, LCD, and Laptop.

3.2.2 Format Selection Result

Format selection for learning tools is adjusted to the principles, characteristics, and steps of the PCL learning approach. The steps of the PCL learning approach consist of opening activity, core activity, and closing activity. Learners' activity sheet, assignment and learning achievement test are made colorful with the expectation of drawing the learners' interest and thus, they would be motivated to learn.

3.2.3 Initial Design Result

In this stage, an initial design of lesson plan for three meetings, the learning media, the learners' activity sheet, learning achievement test for the comparison and chance are made. All the results of this designing phase is called Draft I.

3.3 Description of the Development Stage

3.3.1 Expert Validation Result

Validation and Revision of the Lesson Plan Result (RPP)

The experts validation were focused on the format, content, illustration, and language within the developed learning tools. The result of these experts validation such as, correction, critics, and suggestion were used as basis to revise the lesson plan. The revised lesson plan based on the inputs from the experts is called Draft II. All the three experts gave appropriate and very appropriate grading; they also concluded that the lesson plan could be implemented with minor revision.

Validation and Revision Result of the Learners' Activity Sheet (LKPD)

The assessment made by the validators toward the LKPD consists of: the direction of assignment and information. The experts validation result on the learners' activity sheet turn out to be appropriate and very appropriate result. All three validators concluded that the LKPDcould be used with minor revision. Hence, the LKPD was revised accordingly.

The Validation and Revision Result of the Learning Achievement Test

The assessment made by the validators on the learning achievement test is on the format, language, illustration, and content of the test. The validators gave appropriate and very appropriate remark on the test. All four assessors concluded that the learning achievement test could be used with minor revision. Hence, the learning achievement test was revised accordingly.

3.3.2 Readability Test

Before being implemented, the Draft II has undergone the readability test administered to 6 students in three different junior high schools in Gorontalo province. Those six students were taken from classes that were not the experiments classes. The result of this readability test is called draft III. The input from this test was that there are some misplaced words in LKPD. The revision of the LKPD was then made accordingly to enable the learners to interpret problems or tasks given and to assist them in finding the solutions of those problems.

3.3.3 Learning Tools Trials

The trials were intended to make the developed learning tools flawless, before the learning tools is implemented in experiment classes. The trials were implemented for three meetings, as in the lesson plan. The trials were attended by two observers to observe the learners activities and the teacher's ability in managing the classroom. The teacher's activity observer sat in the back of the class and the students' activity observer sat beside the observed learners. The researcher played the general observer role, to observe the overall process of learning. The data from this trial stage were analyzed, then the result is used as consideration to revise the draft III to make it a good and qualified learning tool.

The data from this trials were data on the activity of the learners, data on the teacher's ability to manage the classroom. Based on the set criteria of teacher's ability to manage the classroom, the teacher's ability in managing the all the three trials classes were under the minimum good category. Meanwhile, the students' activity analysis, it appeared that the percentage of students' activity in each observed aspects in each Lesson plan was in the border of tolerated criteria of ideal time. Therefore, it was concluded that the students' activity was effective.

4. Conclusion and Recommendation

4.1 Conclusion

1. Based on either the topic/content/material aspect of the mathematics subject in junior high school or from all three aspects of PCL approach, several characters values that can be developed, nurtured, and shaped were identified explicitly and implicitly in each math session. Those values are, vigilance, logical thinking, critical thinking, self-confident, tolerant, responsible, communicative, open mindedness, collaboration to learn from each other, take and give, and respect for each other, optimism, ability to make quick and accurate decision, and constant self-introspection.

2. Integration of character education into math learning is done through designing learning activity using the PCL approach that explicitly or implicitly can shape certain character values.

3. PCL is a model of learning approach that requires students mental activity to understand a concept through situation and problem presented in the beginning of the learning session. Knowledge constructed through implementation of learning tools based on PCL contributed to the emergence of character values such as: vigilance, critical, tolerant, communicative, collaboration to learn from each other, take and give, and ability to make quick and accurate decision, and constant self-introspection.

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4.2 Recommendation

1. PCL approach integrated with character education needs to be implemented in math learning in order to develop the learners' character.
2. Further investigation on effectiveness of the developed learning tools especially related to the school character and different social circumstances and different mathematics topic.

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The Effectiveness of Graph Theory's Learning Model Based on Decision-Making System Using Analytical Hierarchy Process (AHP) (Case Study of Semester IV-C Students Academic Year 2014/2015)

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ABSTRACT

Analytical Hierarchy Process (AHP) is one of decision-making system that is highlighted in management science. In AHP technique, the best decision will be derived from several alternative options, by first considering the criteria. In this paper, the author applies the AHP in selecting the best learning model for Graph Theory course. Recommended alternative learning models, are: (1) Direct Learning, (2) Cooperative Learning, (3) Problem Based Learning, (4) Discovery Based Learning, and (5) ICT Based Learning. There are four criteria that need to be considered: (1) Teaching Materials, (2) Lecturer Capability, (3) Students Condition, and (4) Facility. All of alternatives and criteria are observed by researchers for 1 month (8 meetings). From the data analysis, obtained sequence the best learning models based on scoring of the four criteria are: (1) Direct Learning = 0.424, (2) Cooperative Learning = 0.235, (3) ICT Based Learning = 0,142, (4) Discovery Based Learning = 0.110, (5) Problem Based Learning = 0.089. CI = 0.0782 and CR = 0.0869. Because CR ≤ 0.10, so the level of consistency is satisfactory. Furthermore, the decision of AHP (direct learning) was experimented for 1 month (8 meetings). The result of the experiment showed that direct learning model effectively increased students learning achievement in Graph Theory subject.

Keywords: Learning Model, Analytical Hierarchy Process, Graph Theory.

1. Introduction

Learning is defined as a system or process that are planned or designed, implemented, and evaluated systematically so that learners can reach learning goals effectively and efficiently [2]. This is based on the statement in the Law of the Republic of Indonesia on National Education System No. 20 of 2003 article 1, subsection 20 which states that learning is a process of interaction of learners with educators and learning resources in the learning environment [1].

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In mathematics of college, the interaction between lecturer and students in learning process is strongly influenced by the learning model used. As known that, mathematics is a science that requires learners to more abstract thinking, logical, creative, and critical. Therefore, the role of the lecturer as a supervisor is required to use appropriate learning models based on conditions of the learning environment. Not too much to say that learning model that's used by lecturer as half of the soul of learning, because this learning model is the workflow process while learning in the class.

However, mathematics is often a tedious learning among students. Lecturer is often too monotonous in teaching mathematics to students. Whoever, whenever, and wherever the students, as well as any topics, the lecturer assign only one learning model to be applied in classroom. Conversely, some lecturers give a wide variety of learning models, while it is not appropriate with the topic taught. As a result, students become bored, and do not understand the lesson. So that learning objectives are not achieved.

Currently, the mathematical learning models have increased in number and variety. Lecturers can choose one or a combination of several existing learning models. However, in choosing a mathematical model of learning that will be used, lecturers should understand several criteria that include: the condition of the students, teaching materials, lecturer capabilities, and the availability of facilities/learning media. All of these criteria must be analyzed so that the lecturer was able to determine the model of learning which are the most appropriate to used.

In determining the learning model, scientific management systems provide a bid analysis process called Analytical Hierarchy Process(AHP). This process was developed by Thomas L. Saatyand used to make a sequence of decision alternative sand select the best of the available alternatives [3]. So, as a lecture rthat has the logic thinking, mathematics lecturer should perform an analysis process as the first step in determining the most appropriate learning model is based on several criteria that have been mentioned previously.

Therefore, the authors are interested to examine the effectiveness of mathematics learning model based on decision system using Analytical Hierarchy Process(AHP).

2. Material and Method

Analytical Hierarchy Process (AHP) was developed by Dr. Thomas L. Saaty from the Wharton School of Business in 1970, to organize information and judgment in selecting the most preferred alternative. The working principle of AHP is a simplification of a complex problem that is not structured, strategic and dynamic into its parts, and arrange in a hierarchy. Then, the importance level of each variable is given a numerical value subjectively about the significance of these variables relative to the other variables.

Analytical Hierarchy Process (AHP) has many advantages in explaining the decision-making process because it can be described graphically, so it can easily understand by all parties involved in decision-making. Through AHP, complex decision process can be broken down into smaller decisions that can be handled easy. The basic idea of working principle of AHP is:

a) Preparation ofthe hierarchy
The issue that will be resolvedis broken down intoits elements:criteria and alternatives, thenorganized intoa hierarchical structure.

b) Assessment of criteria and alternatives
Standardscale ofpreferencehas been determinedbyDr.ThomasL.Saatytobe usedas areasonable basistocompare twoitems.The standardscale of preferences is shown inthe following table:

Level Preferences	Value
Criteria/alternativeAas important as thecriteria/alternativeB	1
Alittle more importantthanB	3
Aclearly moreimportantthanB	5
Avery clearlymoreimportantthanB	7
Aabsolutelymore importantthanB	9
Whenhesitatingbetweentwoadjacentvalues	2,4,6,8

c) Determining the priorities
For eachcriteria and alternatives, should be paired comparisons. The weightorpriorityis calculated bymatrix manipulationorthrough completion of amathematicalequation.

d) Logicalconsistency
All the elements are grouped logically and consistently grade daccording to a logical criterion below [4]:

n	2	3	4	5	6	7	8	9	10
RI	0	0,58	0,90	1,12	1,24	1,32	1,41	1,45	1,51

The following is a mathematical phase that is used to make decisions based on there commendations of AHP [5]:

a) Develop a pair comparison matrix for each alternative decision based on each criterion.

b) Synthesis:
(1) Add up the value of each column in the pair comparison matrix
(2) Divide the value of each column in the matrix comparison of the pair with the corresponding number of columns (called matrix normalization)
(3) Calculate the average value of each row of the matrix normalization (called vectors preferences)
(4) Combine the preference vector for each criteria into a matrix that shows preference of each alternative based on each criterion.

c) Creating a paired comparison matrix for the criteria.

d) Calculating the matrix normalized by dividing each value in each column of the matrix with a number of related fields.

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e) Creating a preference vectors by calculating the average line on normalization matrix.
f) Calculate the overall score for each alternative decision by multiplying the preference of criteria vector with a criteria matrix.
g) Rank decision alternatives based on the alternatives that have been obtained in the previous step.

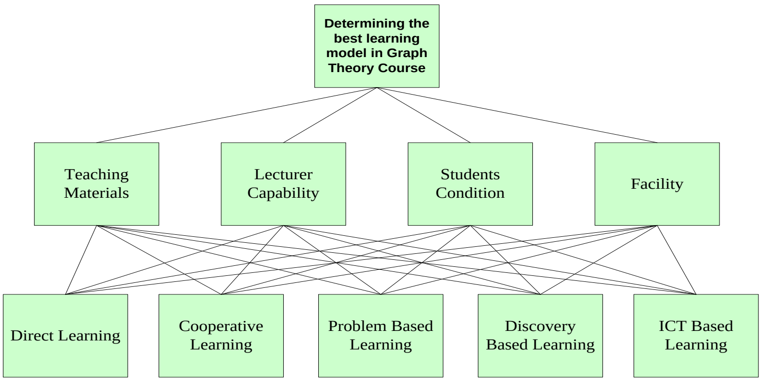
3. Result and Discussion

Application of Analytical Hierarchy Process (AHP)

1. Decomposition of Problem

Determination of the best learning model for Graph Theory subject in 4th semester Department of Mathematics Education IAIN Palopo in this case was done by applying the Analytical Hierarchy Process(AHP). The main purpose(goal) to be achieved is obtaining the best sequence of learning models for Graph Theory subject in 4th semester Department of Mathematics Education IAIN Palopo by applying the Analytical Hierarchy Process(AHP). The learning models recommended are: (1) Direct Learning Model, (2) Cooperative Learning Model, (3) Problem Based Learning Model, (4)Discovery Based Learning Model, and(5) ICT Based Learning Model. While there are four criteria to be considered, are: (1) Teaching materials, (2) Lecturer Capability, (3) Students Condition, and (4) Facility.

Simply, the decomposition of the problem presented in the following diagram:



Picture 1. Hierarchy Structureof AHP for Determining The Best Learning Model

2. Assessment of CriteriaandAlternatives
   a. Criteria assessment

Criteria	Teaching Mat.	Lecturer Cap.	Students Cond.	Facility
Teaching Mat.	1,000	5,000	3,000	7,000
Lecturer Cap.	0,200	1,000	0,200	3,000
Students Cond.	0,333	5,000	1,000	5,000
Facility	0,143	0,333	0,200	1,000

   b. Assessment of learning model based on teaching materials

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Teaching Mat.	Direct	Cooperative	PBL	DBL	ICT
Direct	1,000	3,000	5,000	4,000	7,000
Cooperative	0,333	1,000	3,000	3,000	5,000
PBL	0,200	0,333	1,000	0,333	3,000
DBL	0,250	0,333	3,000	1,000	4,000
ICT	0,143	0,200	0,333	0,250	1,000

c. Assessment of learning model based on lecturer capability

Lecturer Cap.	Direct	Cooperative	PBL	DBL	ICT
Direct	1,000	3,000	5,000	4,000	0,200
Cooperative	0,333	1,000	5,000	3,000	0,167
PBL	0,200	0,200	1,000	0,500	0,125
DBL	0,250	0,333	2,000	1,000	0,143
ICT	5,000	6,000	8,000	7,000	1,000

d. Assessment of learning model based on students condition

Students Cond	Direct	Cooperative	PBL	DBL	ICT
Direct	1,000	2,000	5,000	7,000	3,000
Cooperative	0,500	1,000	3,000	5,000	2,000
PBL	0,200	0,333	1,000	3,000	0,333
DBL	0,143	0,200	0,333	1,000	0,200
ICT	0,333	0,500	3,000	5,000	1,000

e. Assessmen tof learning model based on facility

Facility	Direct	Cooperative	PBL	DBL	ICT
Direct	1,000	2,000	2,000	2,000	5,000
Cooperative	0,500	1,000	2,000	2,000	3,000
PBL	0,500	0,500	1,000	2,000	3,000
DBL	0,500	0,500	0,500	1,000	3,000
ICT	0,200	0,333	0,333	0,333	1,000

3. Preferences Vector

Alternative	Criteria			
	Teaching Mat.	Lecturer Cap.	Students Cond.	Facility
Direct	0,471	0,210	0,431	0,356
Cooperative	0,244	0,136	0,253	0,246
PBL	0,088	0,041	0,090	0,189
DBL	0,152	0,063	0,044	0,145
ICT	0,044	0,550	0,182	0,064

4. Rank each criterion

Criteria	
Teaching Mat.	0,539
Lecturer Cap.	0,110
Students Cond.	0,295
Facility	0,056

5. The result of multiplication of preference vector an dranking criteria

Alternative	Score
Direct	0,424
Cooperative	0,235
PBL	0,089
DBL	0,110
ICT	0,142

6. The order of alternatives based on rankings

Alternative	Score
Direct	0,424
Cooperative	0,235
PBL	0,142
DBL	0,110
ICT	0,089

7. Consistency vector average is 4,2347, so it's obtained the score of CI=0,0782. RI values forn =4is0,9. Thus obtained a score of CR=0,0869. Note that the value of CR≤0,10, so the assessment consistency level is satisfactory.

From the result of AHP for determining the best learning model for Graph Theorys ubject, shows that Direct Learning Modelis the best learning model with ascore of 0,424. Followed by Cooperative Learning Model, ICT Based Learning Model, Discovery Based Learning, and Problem Based Learning Model.

<u>The effectiveness of the AHP's decision (Direct Learning Model) to improve student learning achievement of Graph Theory subject</u>

During8 (eight)meetings, the students are taught by using direct learning model. Furthermore,at the last meeting, lecturer held final-test to measure the results of their study. From 25 students, obtained an averagere sult of their study was 81,96 with the standard deviation of 5,204. Further, analysis is as follows:

a. Hypothesis

1) Descriptive: Direct learning model can improve student learning achievement of Graph Theory over than 75(the minimum value is B).

2) Statistics:

$H_0 : \mu = 75$

$H_1 : \mu > 75$

b. Criteria

Accept H_0 if $t_{counting} \leq t_{tabel}$

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c. T-test

By using theT-test,is obtained:

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{81,96 - 75}{5,204/\sqrt{25}} = 6,686$$

Whilethe value of t_{table} for α = 5%, dk = *n*-1 = 25-1 = 24 is 1,711.

d. Decision

Because of $t_{counting} > t_{table}$, in this case 6,686>1,711,then H_0 rejected and H_1 accepted.It means that: direct learning model can improve student learning achievement of Graph Theory over than 75.

e. Conclusion

Direct learning modelis effective in improving students learning achievement of Graf Theory subject, semester IV students, mathematics education department, academic year 2014/2015.

4. Conclusion

Through the application of Analytical Hierarchy Process (AHP) in determining the best learning model of Graph Theory subject, it's found that direct learning modelis the best choice among 5 alternatives,of course based on pre-established criteria. After learning model directly applicable to the classroom, it was concluded that the model is effective in improving students learning achievement of Graph Theory, semester IV students, mathematics education department,academic year 2014/2015.

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The Application of Connecting, Organizing, Reflecting, and Extending Learning in Enhancing Students' Algebraic Thinking Skill

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ABSTRACT

The aim of this study is to find out the application of Connecting-Reflecting-Organizing-Extending (CORE) learning application in enhancing students' algebraic thinking skill. This study used quasi experiment with non-equivalent control group design. The result shows that CORE learning is more effective in enhancing students' algebraic thinking skill compared to conventional learning. The students' enhancement in algebraic thinking skill from CORE learning group is better than the students from conventional learning group. Students' enhancement in Algebraic thinking skill under CORE learning and conventional learning falls into medium category.

Keyword: Algebraic thinking skill, CORE learning.

1. Introduction

One of study domains or aspects which should be mastered by a student when studying junior/senior high school mathematics is Algebra [1]. Kriegler [2] said that Algebra is a gate to understand mathematics further, since Algebra contains basic concepts of mathematics, such as: sets, functions, and combinatorics. Sets and functions are the basic concept for analysis domain, whereas combinatorial mathematics is the basis for probability and statistics.

Furthermore, Kriegler [2] stated that algebraic thinking should become a guide for learning and teaching mathematics, so that the teacher can assist students in doing mathematics, in order to make them successful. The statement of Kriegler [2] is parallel to Mc Clure [3] who stated that one of approaches to make mathematics curriculum becomes more integrated is by developing students' algebraic thinking in all class levels.

There are two main components in algebraic thinking [2], namely: 1) development of mathematical thinking tools, and 2) study of basic algebra ideas. Mathematical thinking consists of thinking habitual analytically, problem solving skill, reasoning skill, and representing skill. Basic algebra idea is a domain in which

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mathematical thinking can be developed. Mc Clure [3] said that algebraic thinking is a way of thinking which includes analyzing relation among quantity, analyzing structure, learning a change, generalization, problem solving, modeling, justification, proving, and predicting.

Based on the above opinion about algebraic thinking, it can be concluded that algebraic thinking is a basic important element of mathematical thinking and reasoning. On the other hand, however, it is still found that there is problem in students' algebraic thinking skill.

Yumiati [4] said that students experience difficulty when they learn algebra for the first-time in Junior High School. Students who get used to learn arithmetic will easily count if 15 + 20 = 35, then 20 = 35 – 15. But, students who just learn algebra in junior high school could have difficulty when they are given the algebraic expression of "$x + 42 = 77$, then $x = ...$". Yumiati [4] also found student's mistake in defining the form of algebra "$2x$". In algebra $2x$ means $2 \times x$, but there are many students who define that $2x = 20 + x$, such as what was done by a student as follow. When asked to determine the value of function $f(x) = 2x + 3$ for $x = 7, 9, 11$ and 13, a student did it as follows:

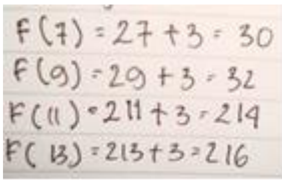

Figure 1. Student's Mistake in Defining Form of Algebra $f(x) = 2x + 3$

The problems which are often found in students' algebraic thinking can be resulted from learning process in class. Based on the importance of algebraic thinking as a gate to understand mathematics further, it is necessary to use this kind of thinking in learning which can enhance students' algebraic thinking skill. According to Bernarz, Kieran and Lee [5], class environment with collaborative learning situation, forces students' discourse, gives opportunity to student in communicating mathematical ideas and conjecture can better facilitate algebraic thinking.

The same problem was revealed by Carpenter, Franke, and Levi [5] that it is important for teacher to facilitate algebraic thinking through meaningful discourse. From those two opinions it can be said that for the development of students' algebraic thinking skill, mathematics learning should be student-centered adopts. Students are given opportunity to actively discuss and communicate in giving their ideas through a discourse. Discourse in mathematics can be a mathematical problem whose answer cannot be directly answered by students. Discourse in mathematics can be a mathematical problem whose answer cannot be directly answered by students.

Yumiati [6] said that CORE learning which follows constructivism principle could be expected to enhance algebraic thinking skill. According to Curwen, et.al.

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[7], CORE learning combines four important elements of constructivism: *connect* to students' initial knowledge, *organize* new material to students, give opportunity to students to *reflect* strategically, and give opportunity to student to *extend* learning.

Stages in CORE learning are: 1) Connecting stage: teacher activates students initial knowledge which is related with material which can be learned and students share each other which is related with that initial knowledge; 2) Organizing stage: student organize the material which is mastered to justify new fact/principle through solving the problem which is given by teacher. Justification is actively created by students with teacher guidance. Organizing the ideas can help students to understand the concept; 3) Reflecting stage: students examine knowledge which is organized and justification, and make a revision; 4) Extending stage: students are given opportunity to synthesis his/her knowledge, organize it by new way and change their knowledge and change it for new application.

The stages in CORE learning are: 1) Connecting stage: teacher activates students by asking them to observe the relationship between the new concept and the initial knowledge related to the new material and the students share the concepts to each other; 2) Organizing stage: students organize the material which is mastered to justify new fact/principle through solving the problem given by the teacher. Justification is actively created by students under teacher guidance. Organizing the ideas can help students to understand the concept; 3) Reflecting stage: students examine knowledge which is organized and justified, and make a revision; 4) Extending stage: students are given opportunity to synthesize his/her knowledge, organize it by new way and change their knowledge and change it for new application.

Yumiati [7] describes interrelation between CORE learning stages and algebraic thinking skill as follow.

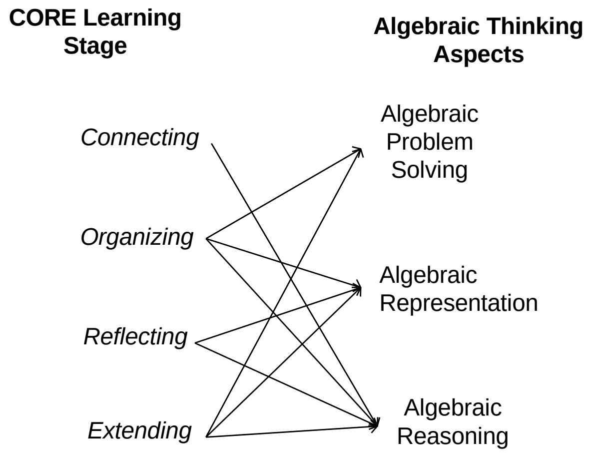

Figure 2. Interrelation of CORE Learning and Algebraic Thinking Skill

This article discusses study result about the application of CORE learning in enhancing students' algebraic thinking skill.

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2. Methodology

This study used quasi experiment study with pretest-posttest non-equivalent group design in which experiment class and control class are not selected randomly [8]. There are two groups of class, namely: experiment group which is taught by using CORE learning and control group which is taught by using conventional learning. Conventional learning or classical learning is learning model which is used to be carried out daily by teacher in which teacher explains learning material, gives example of problem and the way to solve it, gives opportunity to student to ask question, then teacher gives the students problems to be solved as a drill.

The sample of this study were selected randomly from Public Junior High School (PJHS) in North Jakarta City Indonesia. PJHS 30 is selected as a sample, then eighth grader students were selected through purposive sampling technique as sample subject with the following consideration: 1) Eighth grader students were not preparing School Final Exam, thus the research did not disturb their preparation; and 2) Eighth grader students have been more adapted to new school (from Elementary School to Junior High School) compared to eighth grader students. From Eight grade in PJHS 30, two classes were selected randomly to become experiment class and control class. The number of students are 31 students in experiment class and 30 students in control class. In both classes, all students were given pretest (before learning) and posttest (after learning).

The instruments of this study consist of algebraic thinking skill test and observation sheet. Algebraic thinking skill test is in the form of essay consisting of 5 items. Observation sheet was used to get the description about teacher's learning process quality and students' activity during learning process took place. In addition, observation sheet was used to make sure that CORE learning implementation was consistent to the theory.

The data is analyzed qualitatively for application of CORE model and enhancement of students' algebraic thinking skill, and quantitatively to see the comparison of algebraic thinking skill enhancement between students from CORE learning group and students from conventional learning group. The enhancement of students' algebraic thinking skill can be observed from N-gain between two learning groups. N-gain is normalized gain obtained by using the following formulation:

$$\text{N-gain} = \frac{\text{posttest score} - \text{pretest score}}{\text{ideal maximal score} - \text{pretest score}}$$

The criteria of algebraic thinking skill enhancement use N-gain criteria which is suggested by Hake [9] as follow:

Table 1. N-gain Criteria

N-gain criteria	N-gain interval
High	N-gain ≥ 0.7
Medium	$0.3 \leq$ N-gain < 0.7
Low	N-gain < 0.3

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3. Result and Discussion

a. Result

Algebraic Thinking Skill (ATS) Scores which are obtained by students in two learning groups is presented in Figure 2 as follows.

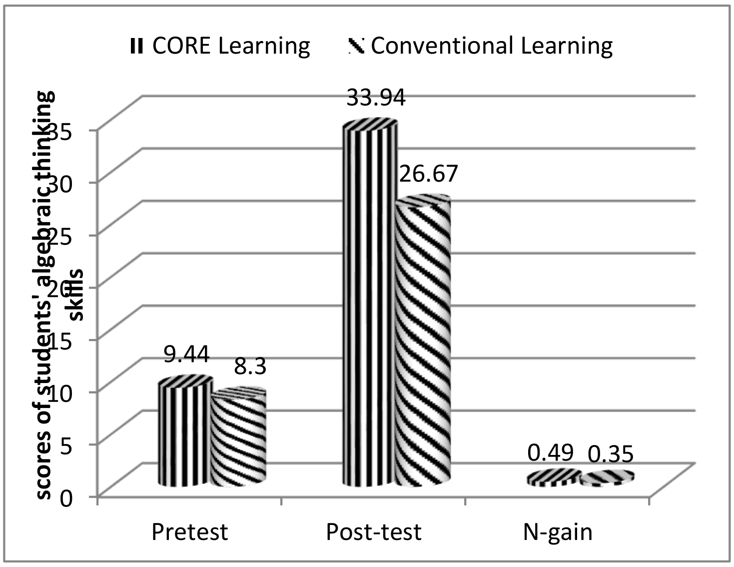

Figure 2. Diagram of Pretest and Posttest Score of students' ATS based on Learning Group

Note: ATS ideal maximal score = 60

Descriptively, it can be seen that ATS pretest mean of students from CORE learning group relatively equal to ATS pretest mean of students from conventional learning group. But, ATS posttest and N-gain (enhancement) of students who worked under CORE learning is higher than students who worked under conventional learning. ATS Achievement (posttest) of students from CORE learning group is above median (30) of ideal maximal score, whereas ATS achievement of students from conventional learning group is below median of ideal maximal score. ATS N-gain of students from CORE learning group and conventional learning group are 0.49 and 0.35, respectively. Both enhancements are in medium category. Descriptively, it can be concluded that CORE learning is more effective in enhancing students' ATS compared to conventional one.

Before the results are analyzed descriptively, a statistic test is required. Statistic test which is used is *t*-test or Mann-Whitney test. To find out which statistic test that is used, normality test is done first by using Kolmogorov-Smirnov test, and variance homogeneity test by using Levene test. The result of students' ATS N-gain normality test is presented in Table 2 as follows.

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Table 2. The Result of Normality Test of Students' Normalized Gain

Group of Data	***N***	**Mean**	**Standard Deviation**	**Kolmogorov-Smirnov Z**	***Sig.* (2-way)**	**H_0**
ATS N-gain of CORE	31	0.49	0.22	0.797	0.548	Accepted
ATS N-gain of Conventional	30	0.35	0.14	0.779	0.579	Accepted

Source: Yumiati [10]

Based on Table 2, it can be concluded that the data of students' N-gain in two learning groups is normal distributed. Furthermore, variance homogeneity test is done toward those groups of data. The result of variance homogeneity test of students' ATS N-gain is presented in Table 3.

Table 3. Result of Variance Homogeneity Test of Students' ATS N-gain Data

Group of Data	**N**	**F**	***Sig.* (2-way)**	**H0**
ATS N-gain of CORE	31	4.62	0.036	Rejected
ATS N-gain of Conventional	30	4.62	0.036	Rejected

Source: Yumiati [10]

Based on the data in Table 3, two groups of data are not homogeneous, thus statistic test which is used is *t*-test by using Separated Variance formulation, namely *t*-test without requires variance homogeneity [11]. Result of *t*-test of students' ATS N-gain is presented in Table 4.

Table 4. The Result of *t*-test of students' ATS N-gain Data

Group of Data	**N**	**Mean**	**Mean Difference**	***t'***	***df***	**Sig. (2-way)**	**H_o**
ATS N-gain of CORE	31	0.49	0.14	3.022	51.44	0.004	Rejected
ATS N-gain of Conventional	30	0.35	0.14	3.022	51.44	0.004	Rejected

Source: Yumiati [10]

The result of *t*-test shows that students' ATS enhancement is different significantly between two learning. Students' ATS enhancement in CORE learning group is higher than students in conventional learning group. Therefore, it can be concluded that CORE learning is more effective in enhancing students' ATS compared to conventional learning.

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b. Discussion

In the beginning of CORE learning application, the students in experiment class looked confused. They were confused about what should do, even though the teacher had already told them about learning scenario. The students who got used to learn by just receiving concept which was given by the teacher, worked hard to think how to invent themselves mathematical concepts through activities: formulating problem, observing, analyzing, drawing conclusion, communicating idea, reflecting and applying. At that time, teacher's patience and perseverance in guiding students are very needed. The teacher should not be in a hurry to present learning material in order that student can implement learning and understand the material given. But this situation only took place in the meeting, once or twice, and subsequently students began to adapt to CORE learning model. This kind of situation in line with the opinion of Brune [12] who said that there is difficulty teacher faced when they implement new methodology into class in which students would not be enough to adapt to their new roles. Changing learning model requires role change for student. Time is needed for them to be able to adapt.

The result of the study shows that CORE learning is more effective in enhancing students' ATS compared to conventional learning. Effectiveness of CORE learning in enhancing ATS can be explained through steps of CORE learning as follows:

1) *Connecting*

In the main activity, the students connect old information to new information and some other concepts. The teacher activates students' initial knowledge which is related to material which will be learned. Through this activity, the teacher can identify student's ability and misunderstanding toward material and gives confirmation. For example, when he/she will discuss Linear Equation material with Gradient sub material. In connecting stage, the teacher connects gradient material to old material namely algebraic forms, Cartesian coordinate, function and linear equation $y = mx + n$. By having question-answer session, student's mistake can be found in dealing with Cartesian coordinate. For example, when a student is asked to determine the coordinate of point P as follows.

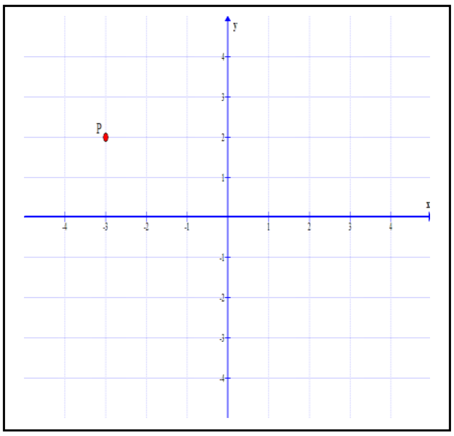

Figure 3. The Location of Point P on Cartesian Coordinate
Source: Yumiati [10]

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It happened that there was a student who determined that the point is (2,-3). Then teacher corrected that mistake. The activity that activated student's initial knowledge can train student's reasoning, because students could apply suitable rule or principle.

2) *Organizing*

This stage is started by giving the problem to students. The problem should be solved by students individually or in a group. The teacher will assist students in the form of scaffolding for those who need it. Problem solving encourages students to think. The problem demanded student to use the knowledge he/she has already possessed to solve that problem and generate knowledge which is new for student.

For example, when students invent Pythagoras formulation inductively and prove that formulation deductively. First, student is asked to determine the length of hypotenuse of several triangles which the length of right angle is known through measurement. From the result of that measurement, a student is asked to conclude what is happened if each side of triangle is squared, then they invent relation of sides in right triangle which is called Pythagoras formulation. The following is an example of student's work result.

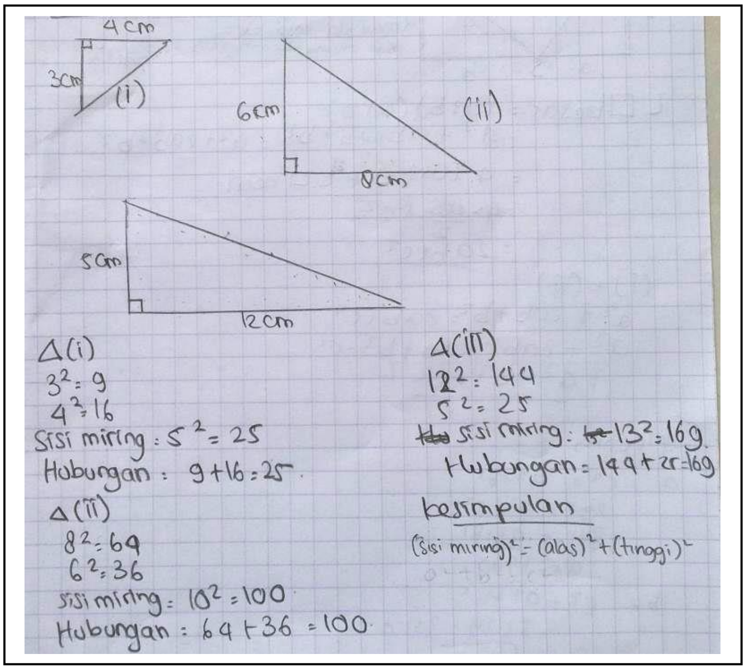

Figure 4. Student's Answer about the Invention of Pythagoras Formulation Inductively
Source: Yumiati [10]

Students obtain Pythagoras formulation:

The length of (hypotenuse)^2 = the length of (base)^2 + the lengh of (height)^2

Pythagoras formulation is obtained by students through inductive thinking pattern, and then it is deductively proved. The following presented one of group's answer about proving of Pythagoras formulation deductively.

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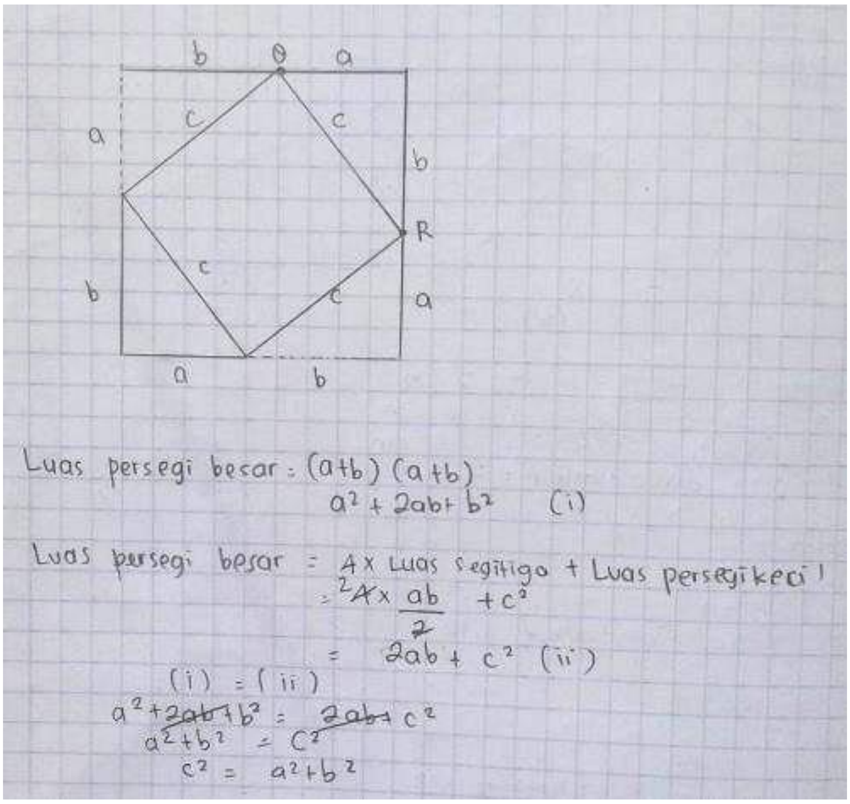

Figure 5. The Student's Answer about Pythagoras Formulation Proving Deductively
Source: Yumiati [10]

In solving a right triangle problem, the students can justify new concept, namely: Pythagoras formulation. Subsequently, the students look for important ideas from initial knowledge which can be used to solve the problem. Important ideas which can be taken are width of square and multiplication of $(a + b)(a + b)$. Those activities need good reasoning, because it contains activity to justify and draw conclusion.

The process of organizing the material will encourage the students to understand the problem given by teacher which is related to new material. The students should choose a strategy which is appropriate to solve the problem, and do problem solving. Problem solving strategy can be different among students. For example when students should solve the following problem.

Sinta will buy chocolate bread and cheese bread. She plans to buy 12 breads.

a. How many each of chocolate bread and cheese bread which is possible to be bought by Sinta?
b. According to you, does the answer for the problem above is single (only one answer) or multiple answers?
c. Show the sum of chocolate breads and cheese breads in form of equation.

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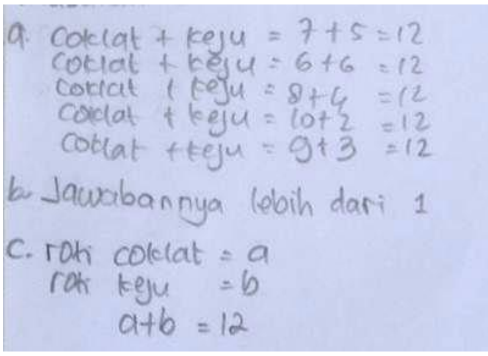
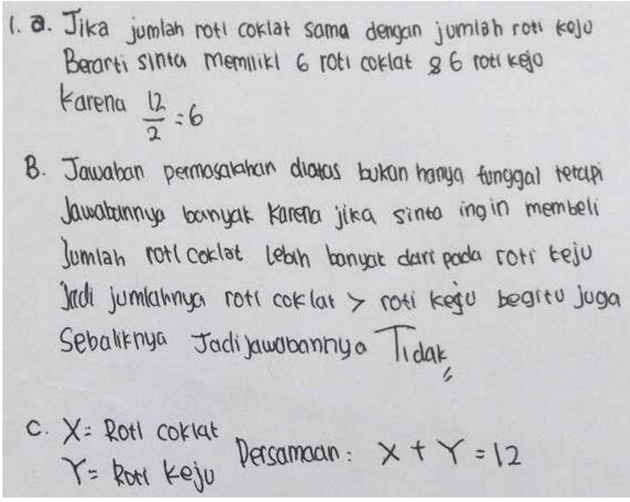
Several different solutions are showed by some groups of students which is presented as follow.


a. Coklat + keju = 7 + 5 = 12
Coklat + keju = 6 + 6 = 12
Coklat + keju = 8 + 4 = 12
Coklat + keju = 10 + 2 = 12
Coklat + keju = 9 + 3 = 12
b. Jawabannya lebih dari 1
c. roti coklat = a
roti keju = b
a + b = 12

Student's Answer 1


1. a. Jika jumlah roti coklat sama dengan jumlah roti keju
Berarti sinta memiliki 6 roti coklat & 6 roti keju
Karena $\frac{12}{2} = 6$
B. Jawaban permasalahan diatas bukan hanya tunggal tetapi jawabannya banyak karena jika sinta ingin membeli jumlah roti coklat lebih banyak dari pada roti keju jadi jumlahnya roti coklat > roti keju begitu juga sebaliknya Jadi jawabannya Tidak
c. X = Roti coklat
Y = Roti keju
Persamaan : X + Y = 12

Student's Answer 2

Figure 6. The Student's Answer about Linear Equation in Two Variables
Source: Yumiati [10]

The examples which was given in Figure 6 show that in CORE learning, a student is given opportunity to use some strategies in solving the problem. As said by Booker and Obligasi [5] that development of algebraic thinking skill can be achieved if student is encouraged to use various strategies in solving the problem. Therefore, activities in CORE learning can enhance students' ATS.

3) *Reflecting*

Reflection is a contemplation about what is just learned and what has been learned before. In this stage, students discuss what is obtained in organizing stage. Students present their work result to be analyzed by other students or other groups. For example, when students should invent the sum of length of the two shorter sides (squared) and longest side (squared) in acute triangles and obtuse triangles. They should show their work result by displaying it in front of class.

In expressing their work result, students explain it in the form of sentences, figures or graphics which is representation form of understanding toward the concept have been learned. In addition, in reflecting stage, teacher also gives material solidification in the form of problems to be done in their group. Solidification activity is also related to reasoning. Therefore, an activity which is done in this reflecting stage can enhance students' algebraic thinking skill in reasoning and representation aspects.

4) *Extending*

Extending is activity to develop, broaden and use new knowledge as deeper application from that new knowledge. Students should show their ability in solving

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Line k passes through point (1,1) and (-3,-7). Line l passes through (2,3) with gradient -1. Determine the intersection point of k and l.

complex problems, related to various topics. Following is an example of problems which is given by ateacher.

The solution of this problem should make use of principles of linear equation which pass through two points, principles of linear equation which pass through a point with gradient known, and principles of linear equation system solution with two variables. The example of student's answer is presented as follows.

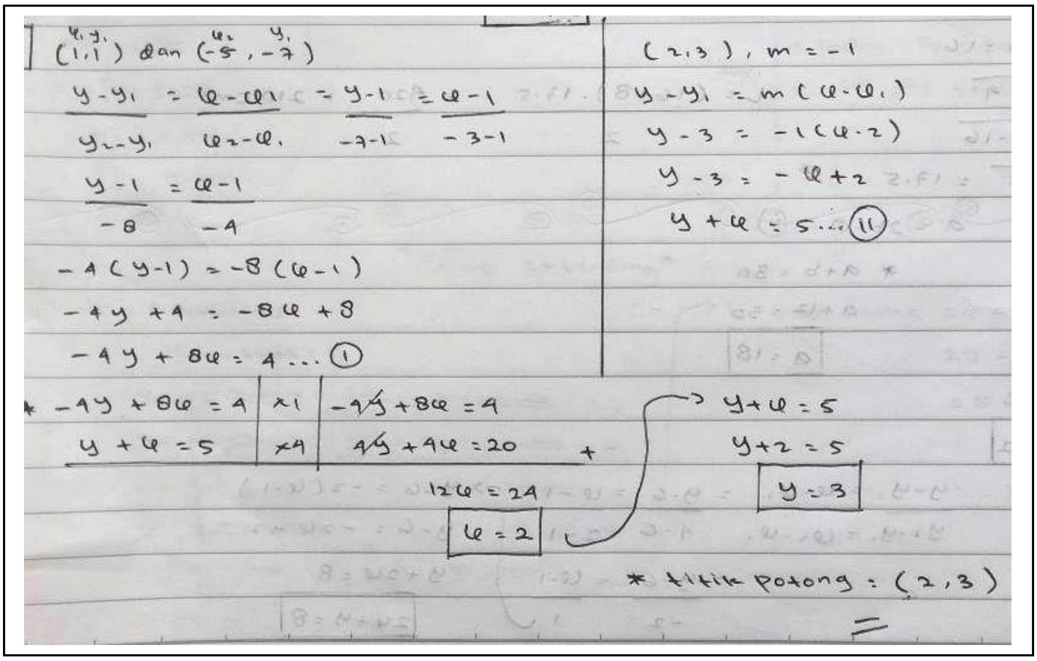

Figure 7. The Student's Answer in Solving Problems in Extending Stage

Source: Yumiati [10]

Students should solve the problem in which the answer is not directly obtained; however, the students need sharper thinking skills. This is related to algebraic problem solving aspect. Students should show through expression of algebra form or pictures, and this is related to algebraic representation aspect. In addition, students should show his/her reasoning in choosing suitable concepts/principles related to problem solving. Therefore, in solving the problem which is given in extending stage, students should use all aspects in algebraic thinking. These activities can impact on students' ATS enhancement on all aspects. Therefore, the habits of the students in each stage of CORE learning in a whole can enhance students' ATS.

The result of study which is related to this study (CORE learning) is in line with the study result of Curwen, et.al. [7], Wijayanti [13], and Azizah [14]. Curwen, et.al. [7] , in which they found that CORE learning model is one of learning models which is effective in developing teacher profession and their metacognition development. Wijayantis' study [13] concluded that students' mathematical problem solving skill in class by using CORE learning is better than class by using conventional learning. Study result of Azizah [14] showed that students in class who worked under CORE learning with constructivism nuance in circle equation concept achieve learning completeness with mean grade of class 73 and there are 87.5% students who exceed limit of minimal completeness criteria grade (in the amount of 70).

Effectiveness of CORE learning in enhancing students' ATS is showed by the existence of significant difference of ATS enhancement between students of CORE learning group and students of conventional learning group. ATS enhancement of students from CORE learning group is higher than those from conventional learning group. This result of study is parallel to the result of Suhaedis's study [15] which concluded that the students' enhancemnet in algebraic thinking skill who worked under constructivism-based learning (Realistic Mathematics Education Approach) is better than those who worked under conventional learning.

4. Conclusion

Conclusions which are obtained in this study are: 1) ATS enhancement of the students from CORE learning group is better significantly from the students from conventional learning group; 2) The amount of the students' ATS enhancement is in medium category in two learning groups; and 3) ATS achievement of the students from CORE learning group is above median of ideal score, whereas ATS achievement of the students from conventional learning is below median of ideal score.

Recommendations which are given as the results of this study are: 1) CORE learning can be used as one of the learning alternatives to enhance students' ATS. Even though there is no learning model which is better to be applied in heterogeneous class situation, but this result of study shows that CORE learning is better than conventional learning in enhancing the students' ATS; 2) The application of CORE learning in enhancing another mathematical ability needs to be studied further.

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