

# Continuous threshold harvesting in a gause-type predatorprey model with fractional-order

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**Submission date:** 25-Apr-2023 12:55AM (UTC+0800)

**Submission ID:** 2074184071

**File name:** 040001\_1\_online.pdf (854.1K)

**Word count:** 3495


**Character count:** 16934

RESEARCH ARTICLE | SEPTEMBER 22 2020

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# Continuous Threshold Harvesting in a Gause-Type Predator-Prey Model with Fractional-Order

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**Abstract.** Harvesting policy is an important issue in maintaining the existence of a population. This paper is focused on studying the effects of continuous predator threshold harvesting policy on the dynamical behavior of a fractional-order Gause-type predator-prey system. This policy is applied to ensure that harvesting does not occur when the population density is less than a specified threshold. The dynamical analysis is done to study the local stability of equilibrium points and the existence of Hopf bifurcation. By using a fractional-order predictor-corrector method, the numerical results are shown to illustrate the analytical result.

**Keywords:** Fractional-Order, Predator-Prey, Threshold Harvesting

## INTRODUCTION

The predator-prey relationship is a dominant topic in ecology and applied mathematics, especially in studying the existence of both populations due to their interaction [1, 2]. The interaction between predator and prey is modeled by assuming that the population density are time-dependent as the result of growth rate, death rate, and predation rate. Suppose that the interaction between predator and prey follows the Gause-type predator-prey model [1, 3], where the prey grows logistically and the predation rate is linear. This model is given by

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{x}{K}\right) - mxy \\ \frac{dy}{dt} &= nxy - dy\end{aligned}\quad (1)$$

where  $x(t)$  and  $y(t)$  represent the density of prey and predator, respectively,  $r$  is the intrinsic growth rate of prey,  $K$  is the environmental carrying capacity of prey,  $m$  is the attack rate of predator on prey,  $n$  is the conversion efficiency of predation, and  $d$  is the natural death rate of predator. On some ecological systems, the population density is threatened due to harvesting, see [4, 5, 6]. This harvesting must be managed to maintain the existence of population. One of the most popular harvesting management is continuous threshold harvesting policy [4, 7] defined by:

$$H(y) = \begin{cases} 0 & , \text{ if } y < T \\ \frac{h(y-T)}{c+(y-T)} & , \text{ if } y \geq T \end{cases} \quad (2)$$

where  $h$ ,  $T$ , and  $c$  are harvesting rate, harvesting threshold, and half saturation constant, respectively. The continuous threshold harvesting means that the harvesting occurs only when the population density is greater than the threshold level  $T$ .

Recently, Lv et.al [4] proposed a Gause-type predator-prey model with Holling type-II functional response and continuous threshold harvesting in predator

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{x}{K}\right) - \frac{mxy}{a+x} \\ \frac{dy}{dt} &= \frac{nxy}{a+x} - dy - H(y).\end{aligned}\quad (3)$$

Note that all the models mentioned above are in the form of a first order differential equation system, which means that the growth rate of each population depends only on the current state. In fact, the growth rate also depends on all previous states (known as memory effects). To include such memory effects, researchers apply a system of fractional differential equations due to the nonlocality of the fractional derivative operator. Some examples of the application of

fractional differential equations for modeling population dynamics can be seen in [8, 9, 10, 11, 12, 13]. In this paper we revisit system (3). Here we assume that the prey and predator growth rates depend on all previous states and prey is available in abundance so that the bilinear functional response is applied. Thus, we arrive at the following system of fractional differential equations

$$\begin{aligned} D_*^\alpha x &= rx \left(1 - \frac{x}{K}\right) - mxy, \\ D_*^\alpha y &= nxy - dy - H(y), \end{aligned} \quad (4)$$

where  $D_*^\alpha$  represents the Caputo fractional derivative of order- $\alpha$  [14] with  $\alpha \in (0, 1]$ , and defined by

$$D_*^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_a^t \frac{f'(\tau)}{(t-\tau)^\alpha} d\tau.$$

As far as we know, the dynamical analysis of model (4) has not been studied. Hence, we propose and investigate a fractional order Gause-type predator-prey model with continuous threshold harvesting policy. In the next section, we discuss the dynamical behavior of the system (4) including the stability of equilibrium point and the existence of Hopf bifurcation. Some numerical simulations that supports the analytical results are also presented.

## EQUILIBRIUM POINT AND LOCAL STABILITY

The locally dynamical behavior that consists of equilibrium point and its stability is defined by

**Theorem 1.** (See [14]) Consider the Caputo fractional differential equation

$$D_*^\alpha \vec{x}(t) = f(\vec{x}(t)), \quad x(0) = x_0, \quad \alpha \in (0, 1], \quad x(t) \in \mathbb{R}^j. \quad (5)$$

The equilibrium point  $\vec{x}^*$  of (5) is local asymptotically stable if all eigenvalues  $\lambda_j$  of its Jacobian matrix  $J = \frac{\partial f}{\partial x}$  evaluated at  $\vec{x}^*$  satisfy  $|\arg(\lambda_j)| > \alpha\pi/2$ .

To analyze the dynamical behavior of system (4), we consider two conditions, namely  $y < T$  and  $y \geq T$ . Based on Theorem 1, the dynamics of system (4) is identified as follow.

### Equilibrium points when $y < T$

In this condition, the equilibrium points are obtained by solving the following equations.

$$\begin{aligned} \left(r - \frac{rx}{K} - my\right)x &= 0, \\ (nx - d)y &= 0. \end{aligned} \quad (6)$$

If  $x = 0$ , equations (6) only has one solution namely the origin  $E_0 = (0, 0)$ . If  $r - \frac{rx}{K} - my = 0$ , then the solution of equations (6) are  $E_1 = (K, 0)$  and  $E_2 = \left(\frac{d}{n}, \frac{r(nK-d)}{mnK}\right)$ . By observing the biological condition, the solution of equations (6) is called equilibrium points of system (4) if  $(x, y) \in \mathbb{R}_+^2$ .

**Theorem 2.** 1. The origin  $E_0 = (0, 0)$  is a saddle point.

2. The predator-free point  $E_1 = (K, 0)$  is local asymptotically stable if  $K < \frac{d}{n}$ .

3. The co-existence point  $E_2 = \left(\frac{d}{n}, \frac{r(nK-d)}{mnK}\right)$  exists if  $K > \frac{d}{n}$  and it is always local asymptotically stable.

*Proof.* 1. By evaluating the Jacobian matrix at the point  $E_0$  we obtain

$$J(E_0) = \begin{bmatrix} r & 0 \\ 0 & -d \end{bmatrix},$$

where its eigenvalues are  $\lambda_1 = r$  and  $\lambda_2 = -d$ . It is clear that  $|\arg(\lambda_1)| = 0 < \frac{\alpha\pi}{2}$  and  $|\arg(\lambda_2)| = \pi > \frac{\alpha\pi}{2}$ . Therefore,  $E_0$  is a saddle-point.

2. The Jacobian matrix evaluated at  $E_1$  is

$$J(E_1) = \begin{bmatrix} -r & -mK \\ 0 & nK - d \end{bmatrix},$$

Where its eigenvalues are  $\lambda_1 = -r$  and  $\lambda_2 = nK - d$ . Notice that if  $K < \frac{d}{n}$  then  $|\arg(\lambda_{1,2})| = \pi > \frac{\alpha\pi}{2}$ .

3. It is clear that if  $0 < \frac{r(nK-d)}{mnK} < T$  then  $E_2 \in \mathbb{R}_+^2$ . For  $E_2 = \left(\frac{d}{n}, \frac{r(nK-d)}{mnK}\right)$ , we obtain the Jacobian matrix define by

$$J(E_2) = \begin{bmatrix} -\frac{dr}{nK} & -\frac{dm}{n} \\ \frac{r(nK-d)}{mK} & 0 \end{bmatrix}.$$

The eigenvalues of  $J(E_2)$  are solutions of characteristic polynomial  $\lambda^2 + \frac{dr}{nK}\lambda + \frac{dr(nK-d)}{nK} = 0$ , namely

$$\lambda_{1,2} = \frac{-dr \pm \sqrt{dr(dr + 4dnK - (2nK)^2)}}{2nK}.$$

If  $d < \frac{(2nK)^2}{r+4nK}$  then  $\operatorname{Re}(\lambda_{1,2}) < 0$ . Hence  $|\arg(\lambda_{1,2})| > \frac{\alpha\pi}{2}$ . If  $d \geq \frac{(2nK)^2}{r+4nK}$ , we have  $\lambda_1 + \lambda_2 = -\frac{dr}{nK} < 0$  and  $\lambda_1 \lambda_2 = \frac{dr(nK-d)}{nK} > 0$ . Hence  $\lambda_{1,2} < 0$  and  $|\arg(\lambda_{1,2})| = \pi > \frac{\alpha\pi}{2}$ . This proves that  $E_2$  is always local asymptotically stable.  $\square$

### Equilibrium points when $y \geq T$

The equilibrium points in this condition are identified by solving the following equations.

$$\begin{aligned} \left[r - \frac{rx}{K} - my\right]x &= 0, \\ nxy - dy - \frac{h(y-T)}{c+(y-T)} &= 0. \end{aligned} \quad (7)$$

If  $x = 0$  then the solution of (7) is not in  $\mathbb{R}_+^2$ . If  $r - \frac{rx}{K} - my = 0$ , then we have the other co-existence equilibrium point which is defined by  $E^* = (x^*, y^*)$  with  $x^* = \frac{(r-my^*)K}{r}$  and  $y^*$  is the real positive solution of the following cubic equation,

$$(y^*)^3 + \frac{dr + cmnK - (mT + r)nK}{mnK}(y^*)^2 + \frac{((nKT + cd + h) - (cnK + dT))r}{mnK}y^* - \frac{hrT}{mnK} = 0.$$

**Theorem 3.** Suppose that:

$$\begin{aligned} \xi_1 &= \frac{rx^*}{K} + d - nx^* - \frac{ch}{(c+y^*-T)^2}, \\ \xi_2 &= mn x^* y^* + \frac{rx^*}{K} \left( d - nx^* - \frac{ch}{(c+y^*-T)^2} \right), \\ \alpha^* &= \frac{2}{\pi} \tan^{-1} \left( \frac{\sqrt{4\xi_2 - \xi_1^2}}{|\xi_1|} \right). \end{aligned}$$

The co-existence point  $E^*$  exists if  $T \leq y^* \leq \frac{r}{m}$ , and it is local asymptotically stable if,

1.  $\xi_1^2 \geq 4\xi_2$ ,  $\xi_1 > 0$  and  $\xi_2 > 0$ , or

$$2. \xi_1^2 < 4\xi_2 \text{ and } \alpha < \alpha^*.$$

*Proof.* If  $T \leq y^* \leq \frac{r}{m}$  then  $x^* \geq 0$  and  $y^* \geq T$ , thus  $E^* \in \mathbb{R}_+^2$ . We compute the Jacobian matrix at the point  $E^*$  and receive

$$J(E^*) = \begin{bmatrix} -\frac{rx^*}{K} & -mx^* \\ ny^* & nx^* - d + \frac{ch}{(c+y^*-T)^2} \end{bmatrix}.$$

This Jacobian matrix gives quadratic polynomial  $\lambda^2 + \xi_1\lambda + \xi_2 = 0$ , thus we get the eigenvalues:  $\lambda_{1,2} = \frac{-\xi_1 \pm \sqrt{\xi_1^2 - 4\xi_2}}{2}$ . Hence, the stability of co-existence point  $E^*$  is achieved if it satisfies:

1. If  $\xi_1^2 \geq 4\xi_2$  then  $\lambda_{1,2} \in \mathbb{R}$ . Further if  $\lambda_{1,2} < 0$  then  $|\arg(\lambda_{1,2})| > \frac{\alpha\pi}{2}$ . This condition holds if  $\lambda_1\lambda_2 = \xi_2 > 0$  and  $\lambda_1 + \lambda_2 = -\xi_1 < 0$ . Thus if  $\xi_1 > 0$  and  $\xi_2 > 0$  then the local asymptotically stability of  $E^*$  is achieved.
2. If  $\xi_1^2 < 4\xi_2$  then  $E^*$  is local asymptotically stable if

$$\begin{aligned} |\arg(\lambda_{1,2})| &> \frac{\alpha\pi}{2} \\ \left| \tan^{-1} \left( \frac{\sqrt{4\xi_2 - \xi_1^2}}{-\xi_1} \right) \right| &> \frac{\alpha\pi}{2} \\ \tan^{-1} \left( \frac{\sqrt{4\xi_2 - \xi_1^2}}{|\xi_1|} \right) &> \frac{\alpha\pi}{2} \\ \frac{2}{\pi} \tan^{-1} \left( \frac{\sqrt{4\xi_2 - \xi_1^2}}{|\xi_1|} \right) &> \alpha \\ \alpha &< \alpha^* \end{aligned}$$

□

## EXISTENCE OF HOPF BIFURCATION

An equilibrium point undergoes a Hopf bifurcation if its stability changes and limit cycle occurs simultaneously when a parameter value is varied. On the integer-order system, an equilibrium point undergoes a Hopf bifurcation if its Jacobian matrix gives a pair of complex conjugate eigenvalues and the sign of its real part changes when such a parameter is varied [30]. Several references stated that this bifurcation also occurs if the order of derivatives is varied [16, 17, 18, 19], as shown in the following theorem.

**Theorem 4.** (See [17, 19]; Existence of Hopf Bifurcation) Consider the following Caputo fractional differential equation

$$D_*^\alpha \vec{x}(t) = f(\vec{x}(t)), \quad \vec{x}(t) \in \mathbb{R}^2, \quad \alpha \in (0, 1]. \quad (8)$$

If an equilibrium point  $\vec{x}^*$  satisfies the following conditions:

- (i) Its Jacobian matrix gives a couple of complex conjugate eigenvalues  $\lambda_{1,2} = a \pm ib$ , with  $\{a, b\} \in \mathbb{R}$  and  $a > 0$ ,

- (ii) There exists  $\alpha^*$  such that  $m(\alpha^*) = \frac{\alpha^*\pi}{2} - |\arg(\lambda_{1,2})| = 0$  and  $\left. \frac{dm(\alpha)}{d\alpha} \right|_{\alpha=\alpha^*} \neq 0$ ,

then the system (8) undergoes a Hopf bifurcation at the equilibrium point  $\vec{x}^*$  when  $\alpha$  crosses the critical value  $\alpha^*$ . Particularly,  $\alpha$  and  $\alpha^*$  are called bifurcation parameter and bifurcation point, respectively.

Notice that when  $\xi_1^2 < 4\xi_2$  and  $\alpha$  passes through a critical value  $\alpha^*$ , the stable co-existence point  $E^*$  changes to unstable where indicated the occurrence of Hopf bifurcation. Therefore, we represent the Hopf bifurcation at the co-existence point  $E^*$  of system (4) by the following theorem.

**Theorem 5.** Suppose that  $a = -\frac{\xi_1}{2}$ ,  $b = \frac{\sqrt{4\xi_2 - \xi_1^2}}{2}$ ,  $\xi_1^2 < 4\xi_2$  and  $\xi_1 < 0$ . A Hopf bifurcation occurs at co-existence point  $E^* = (x^*, y^*)$  when  $\alpha$  crosses  $\alpha^* = \frac{2}{\pi} \tan^{-1} \left( \frac{b}{a} \right)$ .

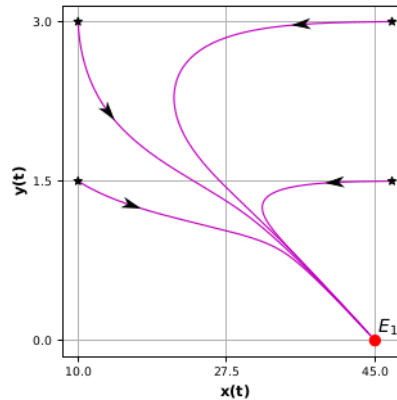
*Proof.* Since  $\xi_1^2 < 4\xi_2$  and  $\xi_1 < 0$ , then we achieve a couple of complex eigenvalues  $\lambda_{1,2} = a \pm ib$  where  $a > 0$ , so that the condition of Theorem 4(i) is satisfied. Now, we determine  $m(\alpha^*)$  and  $\left. \frac{dm(\alpha)}{d\alpha} \right|_{\alpha=\alpha^*}$  to ensure the second condition in Theorem 4.

$$\begin{aligned} m(\alpha^*) &= \frac{\alpha^* \pi}{2} - \min_{1 \leq i \leq 2} |\arg(\lambda_i)| \\ &= \tan^{-1} \left( \frac{\omega}{\theta} \right) - \tan^{-1} \left( \frac{\omega}{\theta} \right) \\ &= 0, \\ \left. \frac{dm(\alpha)}{d\alpha} \right|_{\alpha=\alpha^*} &= \frac{\pi}{2} \neq 0, \end{aligned}$$

thus Theorem 4(ii) holds. According to Theorem 4, a Hopf bifurcation occurs on  $E^* = (x^*, y^*)$  when  $\alpha$  is variated around  $\alpha^*$ . Thus, this theorem is clearly proven.  $\square$

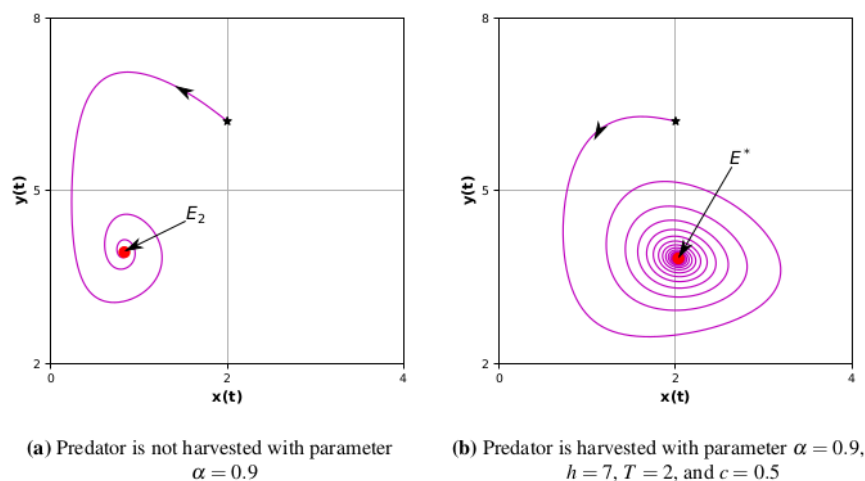
## NUMERICAL SIMULATIONS

To confirm our previous theoretical results, we perform some numerical simulations using the predictor-corrector method for fractional-order differential equations [20]. We first set parameter values  $r = 6$ ,  $K = 45$ ,  $m = 1.5$ ,  $n = 0.02$ ,  $d = 1$  and  $\alpha = 0.9$ . Based on Theorem 2.(ii), we have a saddle point  $E_0$  and the local asymptotically stable equilibrium point  $E_1$  in system (8). This numerical simulation shows that all solutions are going to  $E_1$  which implies the predator will become extinct when  $t \rightarrow \infty$ , see Figure 1.

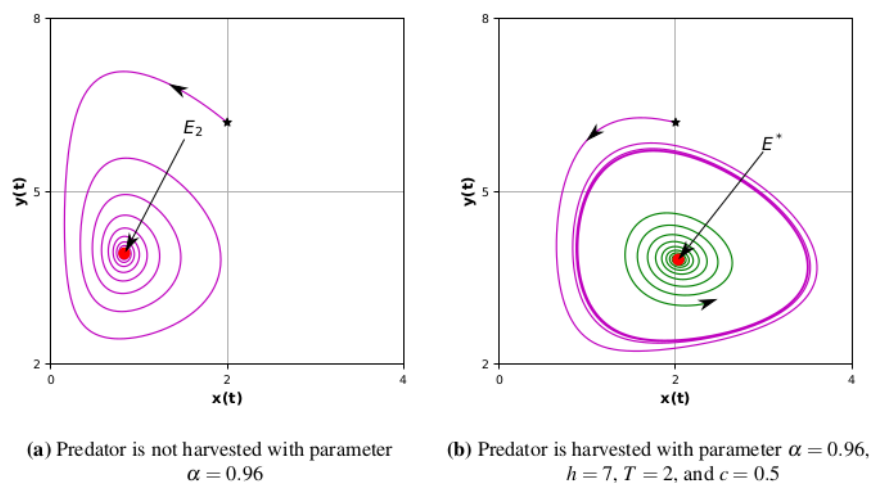


**Figure 1.** The phaseportrait when predator is not harvested by using parameter values  $r = 6$ ,  $K = 45$ ,  $m = 1.5$ ,  $n = 0.02$ ,  $d = 1$  and  $\alpha = 0.9$

Next, we increase the conversion efficiency of predation to the birth rate to  $n = 1.2$ . Here, the equilibrium point  $E_1$  become unstable while the stability of  $E_2$  is achieved, see Figure 2a. Furthermore, if the predator is harvested with parameter  $h = 7$ ,  $T = 2$ , and  $c = 0.5$  then the stability of the co-existence equilibrium point is still maintained. It is



**Figure 2.** The phaseportrait by using parameter values  $r = 6$ ,  $K = 45$ ,  $m = 1.5$ , and  $d = 1$ .



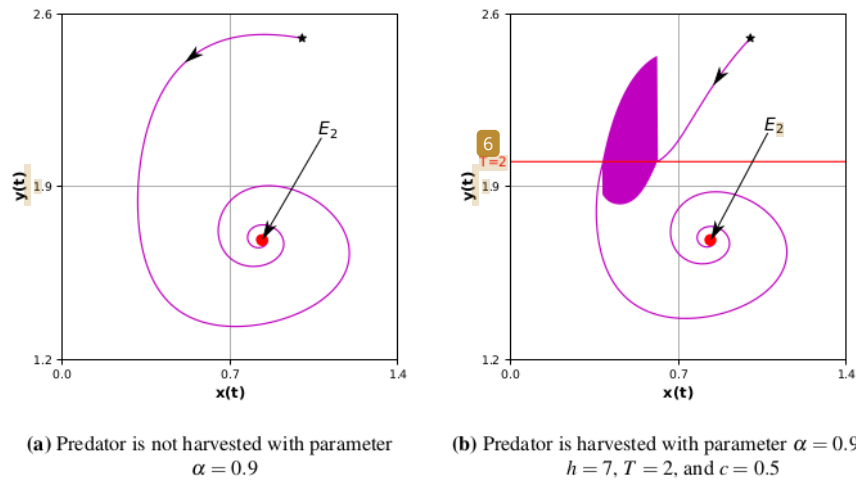
**Figure 3.** The phaseportraits by using parameter values  $r = 6$ ,  $K = 45$ ,  $m = 1.5$ , and  $d = 1$ .

shown that the harvesting doesn't significantly influence the density of predators, but only increases the density of prey, see Figure 2b.

To see the impact of the order of the derivative, we increase the derivative order to  $\alpha = 0.96$ . Based on Theorem 2.(iii), the co-existence point of the model without harvesting on the predator is still stable (Figure 3a). However, according to Theorem 3,  $E^*$  becomes unstable focus. This condition also shows the Hopf bifurcation occurs. According to Theorem 5, the local asymptotically stable co-existence point  $E^*$  (Figure 2b) changes to unstable equilibrium point (Figure 3b) when  $\alpha$  passes through  $\alpha_* \approx 0.95566$ , and continue oscillating as  $t \rightarrow \infty$ . It is also shown that the stable limit-cycle appears and isolates the co-existence point  $E^*$ .

Finally, if we increase the attack rate to  $m = 3.5$ , based on Theorem 3, there exists an asymptotically stable co-existence point when the predator is not harvested (Figure 4a). Furthermore, when the predator is harvested, the co-existence point exists only below the threshold level. For  $t \rightarrow \infty$ , If the predator density above the threshold level,





**Figure 4.** The phase portraits by using parameter values  $r = 6$ ,  $K = 45$ ,  $m = 3.5$ , and  $d = 1$ .

its density decreases and when crosses the threshold level, it is oscillating and then goes to the co-existence point  $E_2$ . It is showed that although the population density of predator is decreasing causes by harvesting, the predator density is maintained due to the threshold predator harvesting, see Figure 4b.

## CONCLUSIONS

The impact of continuous predator threshold harvesting in a Caputo fractional predator-prey model has been studied. It is analyzed that this model has two possible conditions of the equilibrium points, which are three equilibrium points when the predator density is below the threshold harvesting and a co-existence point when predator density is above the threshold harvesting. When the carrying capacity of prey is larger than the ratio of death rate and predation conversion rate of predator ( $K > \frac{d}{n}$ ), both species able to maintain the existence of their population. We also show numerically that the solution around the co-existence point in the harvested system (4) is oscillating through the time, thus the existence of population is still maintained. In the end, we have shown that the population density of predator is still maintained due to the threshold harvesting policy.

## ACKNOWLEDGEMENTS

This research is funded by FMIPA via PNBP-University of Brawijaya according to DIPA-UB No. DIPA-042.01.2.400919/2018, under contract No. 14/UN10.F09/PN/2019.

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