Impact of Fear and Strong Allee Effects on the Dynamics of a Fractional-Order Rosenzweig-MacArthur Model

by Hasan S. Panigoro

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Hasan S. Panigoro and Emli Rahmi

Abstract This paper discusses the impact of fear and strong Allee on the dynamical behaviors of the prey and predator relationship following the Rosenzweig-MacArthur model using fractional-order derivative as the operator. As results, four equilibrium points are identified namely the origin point, a pair of axial points, and the interior point. The origin is always locally asymptotically stable while others are conditionally asymptotically stable. The occurrence of transcritical bifurcation around the axial and Hopf bifurcation in the interior are also successfully investigated. The numerical simulations are conducted to support analytical findings. Some interesting dynamics such as forward bifurcation and bistability condition are also provided numerically.

Keywords Fractional-order · Rosenzweig-MacArthur · Allee effect · Fear effect

1 Introduction

Food chain schemes are always found in nature. Every organism may become a predator to others due to its need for food. As a result, each organism has a chance to go extinct as an impact of this ecological mechanism. Therefore, studying the existence of organisms that have prey and predator relationship always be a crucial issue for researchers. One of the much-publicized ways is using mathematical modeling.

In 1963, a mathematical model is developed by Rosenzweig and MacArthur based on the Lotka-Volterra predator-prey model which assumes that the population of prey grows logistically and its hunting by the predator for foods following Holling type-II as the predator functional response [1]. Nowadays, the Rosenzweig-MacArthur model becomes an attractive reference to establish a novel predator-prey model by

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Table 1 The biological interpretation of variables and parameters

Variables and parameters	Biological interpretation
x	The density of prey
y	Density of predator
r	Intrinsic growth rate of prey
k	Level of fear
K	Environmental carrying capacity of prey
b	Allee threshold
m	Predation rate
a	Half saturation constant of predation
n	Predator growth rate which converted from the predation process
d	Predator death rate

involving some ecological components associated with real phenomena in nature. For example, see [2, 3] and references therein.

In this paper, we assume that the growth rate of prey is influenced by the indirect impact of the predator through the fear effect [4]. We also assume that this intrinsic growth rate could also decrease by the intraspecific competition and difficulty in finding mates is known as the Allee effect [5]. Thus, we have the following model.

$$\begin{aligned} \frac{dx}{dt} &= \frac{rx}{1+ky}\left(1-\frac{x}{K}\right)(x-b)-\frac{mxy}{a+x}, \\ \frac{dy}{dt} &= \frac{nxy}{a+x}-dy. \end{aligned} \tag{1}$$

See Table 1 for the biological interpretation of variables and parameters. The term $(x-b)$ represents the Allee effect where for $b \leq 0$ called weak Allee effect and $b > 0$ called strong Allee effect. In our work, we assume that the intrinsic growth rate of prey affected by strong Allee effect. Due to biological purpose, other parameters also positive constant and both $x(t)$ and $y(t)$ satisfy $(x, y) \in \mathbb{R}^2_+$ where $\mathbb{R}^2_+ := \{(x, y) \mid x \geq 0,\ y \geq 0,\ x \in \mathbb{R},\ y \in \mathbb{R}\}$.

Since the current state of both prey and predator depends on all of their previous conditions, using fractional-order derivative is considered more appropriate in expressing the model better than classical integer-order derivative [3, 6, 7]. Following a similar way with [3, 7] such as replacing the first-order with fractional-order derivative and scaling the time dimension, we obtain the new model as follows.

$$\begin{aligned} {}^{C}\mathcal{D}_t^{\alpha} x &= \frac{rx}{1+ky}\left(1-\frac{x}{K}\right)(x-b)-\frac{mxy}{a+x}, \\ {}^{C}\mathcal{D}_t^{\alpha} y &= \frac{nxy}{a+x}-dy, \end{aligned} \tag{2}$$

where ${}^C\mathcal{D}_t^\alpha$ is Caputo fractional-order derivative defined by

$$ {}^C\mathcal{D}_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_a^t \frac{f'(\tau)}{(t-\tau)^\alpha} \, d\tau, $$

$\alpha \in (0, 1]$ is the order of the derivative and $\Gamma(\cdot)$ is Euler Gamma function [8].

In Sasmal [9], the predator-prey model involving fear and Allee effects has been studied. Sasmal's model is quite similar to ours both in assumptions and the deterministic model. The big difference which becomes the novelty of our works lies in the predator functional response and the operator of the model. In our works, the Michaelis-Menten type is used as the predator functional response which is considered more realistic than bilinear ones. The fractional-order derivative is also used to replace the first-order derivative as the operator to cover the memory effect.

The rest of the paper is arranged as follows. In Sect. 2, the feasibility and local stability of equilibrium points are verified. Furthermore, the existence of transcritical and Hopf bifurcations are examined in Sect. 3. Several numerical simulations are explored in Sect. 4 not only to support the analytical findings but also to show other dynamical behaviors such as the occurrence of forward bifurcation and bistability conditions. We finally end our work by giving a conclusion in Sect. 5.

2 Feasibility and Stability of Equilibrium Points

The feasible equilibrium points of model (2) are acquired by finding the the positive solution of the following equations.

$$ \begin{aligned} \left[\frac{r(x-b)}{1+ky}\left(1-\frac{x}{K}\right) - \frac{my}{a+x}\right] x &= 0, \\ \left[\frac{nx}{a+x} - d\right] y &= 0. \end{aligned} $$

Therefore, four equilibrium points are identified as follows.

(i) The origin $E_0 = (0, 0)$ which represents the extinction of both populations.
(ii) A pair of axial points $E_1 = (b, 0)$ and $E_2 = (K, 0)$ which represent the existence of prey and the extinction of predator.
(iii) The interior point $E_3 = (\hat{x}, \hat{y})$ which represents the existence of both populations where $\hat{x} = \frac{ad}{n-d}$ and $\hat{y}$ is the positive solution respect to y of the following equation.

$$ y^2 + \frac{y}{k} + \frac{\hat{m}}{4k^2 m} = 0, \tag{3} $$

where $3\hat{m} = \frac{4(\hat{x}-K)(\hat{x}-b)(\hat{x}+a)kr}{K}$. Since $E_i \in \mathbb{R}_+^2\ \forall i = 0, 1, 2$, then they always exist. Furthermore, the existence condition of E_3 is given by the following theorem.

Theorem 1 *If $n > d$ and (i) $m \leq \hat{m}$ then the interior point does not exist; (ii) $m > \hat{m}$ then there exists an interior point.*

Proof Since $n > d$ then $\hat{x}$ is always positive. Thus, the existence of E_3 depends on the positive solution of quadratic equation (3). If $m < \hat{m}$ then the solution of equation (3) is a pair of complex conjugate numbers and hence the interior point does not exist. When $m = \hat{m}$, we have $\hat{y} = -\frac{1}{2k} < 0$, and hence E_3 also does not exist. For $m > \hat{m}$, the only positive solution of equation (3) is given by $\hat{y} = -\frac{1}{2k}\left(1 - \sqrt{1 - \frac{\hat{m}}{m}}\right)$. This completes the proof. □

Now, we discuss the local stability for each equilibrium point. The following theorems are presented.

Theorem 2 *The origin $E_0 = (0, 0)$ is always locally asymptotically stable.*

Proof The linearization around E_0 gives the Jacobian matrix as follows.

$$\mathcal{J}(x, y)|_{E_0} = \begin{bmatrix} -br & 0 \\ 0 & -d \end{bmatrix}.$$

The eigenvalues of $\mathcal{J}(x, y)|_{E_0}$ are $\lambda_1 = -br$ and $\lambda_2 = -d$ which give $|\arg(\lambda_i)| = \pi > \alpha\pi/2 \; \forall \, i = 1, 2$. According to the Matignon condition [10], E_0 is always locally asymptotically stable. □

Theorem 3 *The axial point $E_1 = (b, 0)$ is locally asymptotically stable if $b > K$ and $n < \frac{(a+b)d}{b}$.*

Proof For the axial point E_1, we have the Jacobian matrix

$$\mathcal{J}(x, y)|_{E_1} = \begin{bmatrix} -\frac{(b-K)br}{K} & -\frac{bm}{a+b} \\ 0 & \frac{bn}{a+b} - d \end{bmatrix}, \tag{4}$$

which give eigenvalues $\lambda_1 = -\frac{(b-K)br}{K}$ and $\lambda_2 = \frac{bn}{a+b} - d$. Based on Matignon condition [10], the local asymptotic stability condition are satisfied when $\lambda_i < 0, \; i = 1, 2$ which are given by $b > K$ and $n < \frac{(a+b)d}{b}$. □

Theorem 4 *The axial point $E_2 = (K, 0)$ is locally asymptotically stable if $b < K$ and $n < \frac{(a+K)d}{K}$.*

Proof The Jacobian matrix evaluated at E_2 is given by

$$\mathcal{J}(x, y)|_{E_2} = \begin{bmatrix} (b-K)r & -\frac{mK}{a+K} \\ 0 & \frac{nK}{a+K} - d \end{bmatrix}, \tag{5}$$

where the eigenvalues are $\lambda_1 = (b-K)r$ and $\lambda_2 = \frac{nK}{a+K} - d$. If $b < K$ and $n < \frac{(a+K)d}{K}$ then $|\arg(\lambda_i)| = \pi > \alpha\pi/2, \;\; i = 1, 2$ that obeys the Matignon condition [10]. □

Theorem 5 *The interior point* $E_3 = (\hat{x}, \hat{y})$ *is locally asymptotically stable if (i)* $\xi_1 < 0$*, or (ii)* $\xi_1 > 0$, $\xi_1^2 < 4\xi_2$*, and* $\alpha < \hat{\alpha}$*, where* $\xi_1 = -\frac{(3\hat{x}^2 - 2(b+K)\hat{x} + bK)r}{(1+k\hat{y})K} - \frac{am\hat{y}}{(a+\hat{x})^2}$, $\xi_2 = \frac{(1+2k\hat{y})ad^3m\hat{y}}{(1+k\hat{y})n^2\hat{x}^2}$*, and* $\hat{\alpha} = \frac{2}{\pi}\tan^{-1}\left(\frac{\sqrt{4\xi_2 - \xi_1^2}}{\xi_1}\right)$.

Proof At E_3, we have

$$\mathcal{J}(x, y)|_{E_3} = \begin{bmatrix} \xi_1 & -\frac{n\xi_2\hat{x}^2}{ad^2\hat{y}} \\ \frac{ad^2\hat{y}}{n\hat{x}^2} & 0 \end{bmatrix}. \tag{6}$$

Therefore, the polynomial characteristic is obtained as follows.

$$\lambda^2 - \xi_1\lambda + \xi_2 = 0. \tag{7}$$

Since $\xi_2 > 0$, by obeying Proposition 1 in [11], the stability conditions given in Theorem 5 are proven. □

3 Bifurcation Analysis

In this section, we give two types of bifurcations phenomena namely transcritical and Hopf bifurcations by following theorems.

Theorem 6 *Suppose that* $n < \min\left\{\frac{(a+b)d}{b}, \frac{(a+K)d}{K}\right\}$*. Two axial points* E_1 *and* E_2 *exchange their stability via transcritical bifurcation when* b *crosses* K*.*

Proof Since $n < \min\left\{\frac{(a+b)d}{b}, \frac{(a+K)d}{K}\right\}$, we have $|\arg(\lambda_2)| = \pi > \alpha\pi/2$ for each Jacobian matrix (4) and (5). Therefore, the stability of E_1 and E_2 depend on the sign of λ_1. When $b < K$, $|\arg(\lambda_1)| = \pi > \alpha\pi/2$ for Jacobian matrix (5) and $|\arg(\lambda_1)| = 0 < \alpha\pi/2$ for Jacobian matrix (4). Hence, E_1 is a saddle point while E_2 is locally asymptotically stable. When $b = K$, $E_1 = E_2$ and $|\arg(\lambda_1)| = \alpha\pi/2$ which represents a non-hyperbolic equilibrium point. For $b > K$ the sign of $|\arg(\lambda_1)|$ for Jacobian matrices (4) and (5) are switched which indicates the stability of E_1 and E_2 changes. According to those circumstances, the transcritical bifurcation occurs driven by the Allee threshold (b). □

Theorem 7 *Let* $\xi_1 > 0$ *and* $\xi_1^2 < 4\xi_2$*. A Hopf bifurcation occurs around the interior point* $E_3 = (\hat{x}, \hat{y})$ *when* α *passes through* $\hat{\alpha}$*.*

Proof From (7), the appropriated eigenvalues are given by

$$\lambda_{1,2} = \frac{1}{2}\left(\xi_1 \pm \sqrt{\xi_1^2 - 4\xi_2}\right). \tag{8}$$

Since $\xi_1 > 0$ and $\xi_1^2 < 4\xi_2$, the eigenvalues (8) are a pair of complex conjugate numbers with positive real parts. It is also valid that $m(\hat{\alpha}) = \hat{\alpha}\pi/2 - \min_{1 \le i \le 2} |\arg(\lambda_i)| = 0$ and $\left.\frac{dm(\alpha)}{d\alpha}\right|_{\alpha=\hat{\alpha}} \neq 0$. According to Theorem 4.6 in [12], Hopf bifurcation occurs around E_3 driven by α with $\hat{\alpha}$ is the critical point. □

4 Numerical Simulation

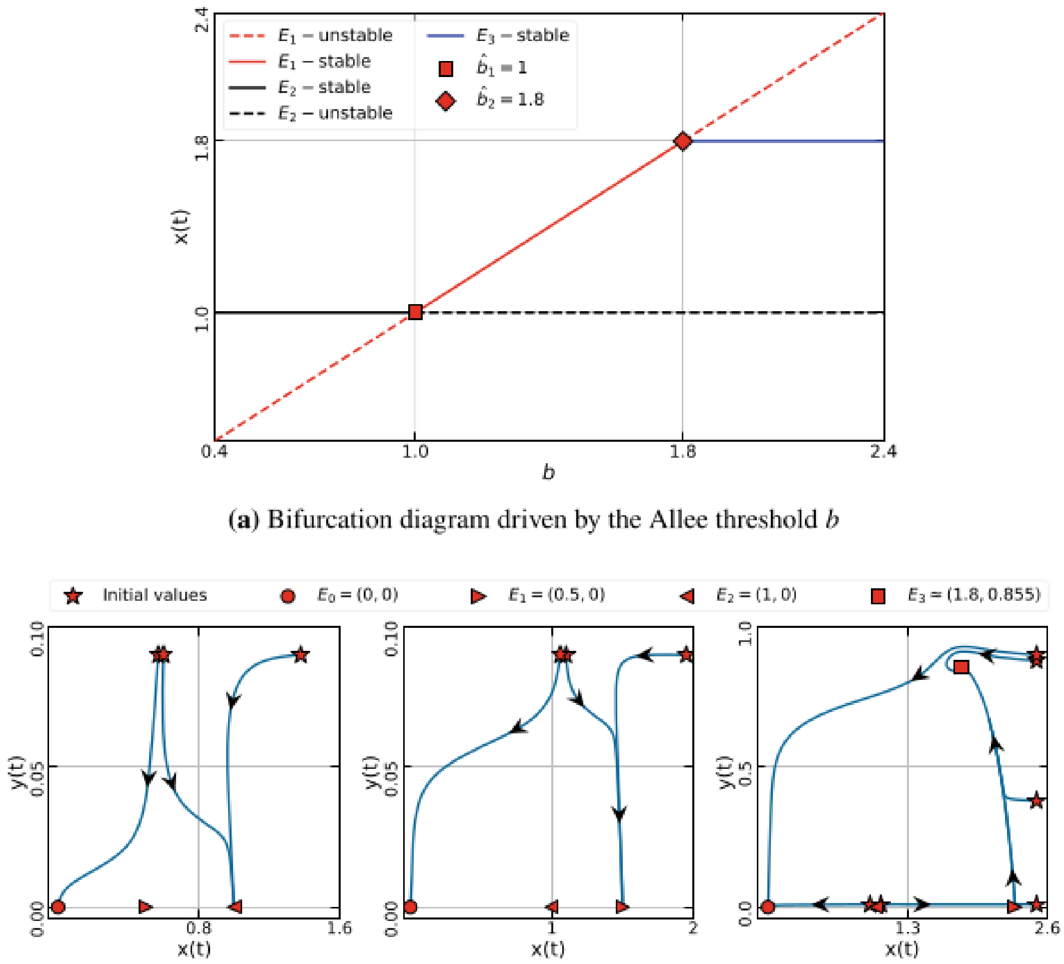
Some numerical simulations are demonstrated using a generalized predictor-corrector scheme given by Diethelm et al. [13]. This scheme is applied to numerical software called *Python-3* to produce some figures such as bifurcation diagrams and time series. In this paper, we study numerically the influence of the Allee threshold (b) and the order of the derivative (α) to the dynamical behaviors of model (2). Since the model does not discuss a specific case, all parameter values are chosen hypothetically by considering the previous analytical results. We first set the parameter as in Table 2 and varying the Allee threshold (b) in interval [0.4, 2.4], see Fig. 1.

From the bifurcation diagram given by Fig. 1a, when b is varied in the interval [0.4, 2.4], the dynamical behaviors change two times. For $0.4 \le b < 1$, we have a locally asymptotically stable equilibrium point E_2 and an unstable point E_1. The stability of both E_1 and E_2 change sign when b crosses $\hat{b}_1 = 1$ which confirm the existence of transcritical bifurcation given by Theorem 6. This dynamical behaviors are maintained for $\hat{b}_1 < b < \hat{b}_2 = 1.8$. Denote that the interior point E_3 does not exist for interval $0.4 \le b < \hat{b}_2$. When b passes $\hat{b}_2$, the axial point E_1 again losses its stability, and a locally asymptotically stable point E_3 emerges which indicates the existence of forward bifurcation. This conditions are preserved for $\hat{b}_2 < b \le 2.4$. Remember that E_0 is always locally asymptotically stable and hence the bistability condition always occurs for each case when the dynamical behaviors change. We perform the phase portraits by picking the values of $b = 0.5, 1.5, 2.3$, which presents the dynamical behavior for each interval. See Fig. 1b, c, d. The stability shifts from E_2 to E_1 and finally to E_3 while E_0 always locally asymptotically stable. This means, the bistability condition always exists for [0.4, 2.4] except in every bifurcation point. This means that the existence of populations depends on the initial values. From those phase portraits, we show that for the given two close initial values, the solutions tend to distinct equilibrium points. Both populations could be extinct or only the existence of prey is preserved.

The next simulation aims to show the influence of the order of the derivative (α) to the dynamical behaviors of model (2). The parameter values are chosen as

Table 2 Parameter values for numerical simulations given in Fig. 2

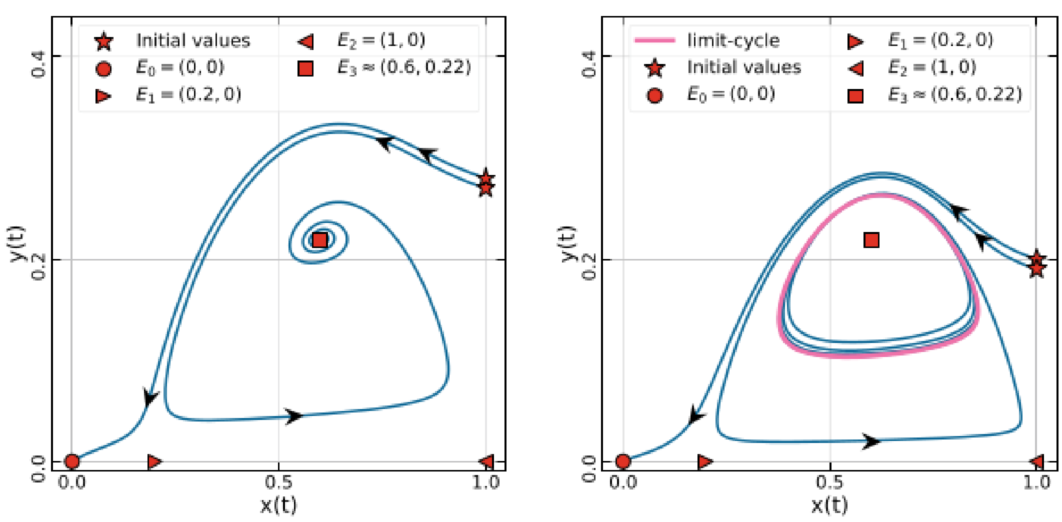
Parameters	r	k	K	m	a	n	d	α
Values	0.4	0.8	1	0.3	0.9	0.15	0.1	0.9



(a) Bifurcation diagram driven by the Allee threshold b

(b) $b = 0.5$ **(c)** $b = 1.5$ **(d)** $b = 2.3$

Fig. 1 Bifurcation diagram and phase portraits of model (2) with parameter values as in Table 2



(a) $\alpha = 0.8$ **(b)** $\alpha = 0.9$

Fig. 2 Phase portraits of model (2) with parameter values as in Table 3

Table 3 Parameter values for numerical simulations given in Fig. 2

Parameters	r	k	K	b	m	a	n	d
Values	0.4	0.8	1	0.2	0.3	0.6	0.2	0.1

in Table 2. Based on Theorem 5, The Jacobian matrix (6) has a pair of complex conjugate eigenvalues with positive real parts. Thus, from Theorem 7, the interior point E_3 undergoes a Hopf bifurcation when α passes through the critical point $\hat{b}$. By using these parameter values, we confirm that the critical point is $\hat{\alpha} \approxeq 0.84304$. To show this condition, we pick $\alpha = 0.8$ and $\alpha = 0.9$ and the numerical results given by the phase portraits in Fig. 2. When $\alpha = 0.8$, two locally asymptotically stable equilibrium points occur i.e. $E_0 = (0, 0)$ and $E_3 \approx (0.6, 0.22)$. As the impact, the model (2) leads to bistability condition. For two close initial values, the solutions convergent to different equilibrium points namely E_0 and E_3. When α is increased to 0.9, E_3 losses its stability and nearby solution convergent to a periodic signal namely limit-cycle. Although the interior point is unstable, both populations are still preserved periodically around the interior point. This ends our numerical simulations.

5 Conclusion

The dynamical behaviors of a fractional-order Rosenzweig-MacArthur model involving fear and strong Allee effects have been studied. The model has four equilibrium points namely the origin, a pair in axial, and a unique interior point. Those two equilibrium points in the axial may exchange their stability via transcritical bifurcation. For the interior point, the stability may change via Hopf bifurcation driven by the order of the derivative. To support the analytical findings, numerical simulations are provided including a bifurcation diagram and phase portraits. We have found numerically that the model undergoes transcritical bifurcation, forward bifurcation, Hopf bifurcation, and bistability conditions. From the biological viewpoint, these circumstances mean that the existence of both prey and predator are threatened due to predation mechanism, fear, and allee effects.

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