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Ability estimation in computerized adaptive test using Mamdani Fuzzy Inference System

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Abstract. Assessment is an activity to find out the learning outcomes of a course. Conventionally, assessment is done using a pencil and paper test. Along with the development of information and communication technology, evaluation can be done computerized, known as Computer Based Test (CBT). The Computerized Adaptive Test (CAT) is one form of CBT where the items given are chosen based on the students' abilities. This research aims to design a system that can estimate the ability of students based on the parameters of the questions and answers given. The estimation method uses the Mamdani Fuzzy Inference System (MFIS). The input MFIS is a level of difficulty and discrimination of the questions, the probability of students being able to answer correctly, and the student's answer, while the output is an ability estimated. Based on this fuzzy system output, the next item questions will be determined according to the ability of the students. 24 IF-THEN rules are used for fuzzy systems. CAT simulations are carried out for Linear Algebra Course with six topics, namely vector, matrices concepts, matrices operation, determinant, inverse, and matrices applications. The type of question given is multiple choice. Giving items will be stopped if the value of the estimated ability of the students has not changed. From the simulations carried out, for each topic, the ability of the student can be acquired with the number of questions as many as six questions. So that the CAT system can minimize the time of the exam, reduce the subjectivity of the assessment and can arrange for each student to get the questions according to his abilities.

1. Introduction

Assessment activity of learning outcomes in the cognitive/knowledge domain is usually done with an objective and descriptions test techniques with various types and carried out conventionally using paper or also called a pencil and paper test. Along with the development of information and communication technology, the assessment of student learning outcomes can be done computerized (known as Computer Based Test/CBT). The use of CBT in examinations can reduce the subjectivity of lecturers in conducting assessments, is not affected by the condition of the lecturers (tired, unfocused), and certainly can reduce the use of paper (following environmentally friendly principles). At CBT, it is possible to randomize questions between one student and another student. In the sense that every student is allowed to get a different sequence of questions. This can prevent students from collaborating in completing the exam. However, the form of an exam like this can cause inaccuracy in giving questions and inefficient time between one student and another.

Computerized Adaptive Testing (CAT) is a test method for test participants that is managed based on the level of participants' abilities [1]. Based on that, the score of the assessment of each examinee



will not be much different, both those with more and moderate abilities. This is because the system used in The CAT can recognize the level of participants' abilities so that the questions given are a matter that can be done by the participants. The design of the CAT as an instrument for evaluating learning outcomes has advantages including (a) assisting lecturers in evaluating learning outcomes (reducing subjective elements, not affected by lecturers' condition/tired or out of focus); (b) in accordance with environmentally friendly principles because it reduces the use of paper in conventional examinations; (c) can be integrated with online learning (e-learning) so that it allows students to conduct self-assessment.

The primary key to the CAT program is the computer's ability to provide the right questions adaptively, according to the estimation of test-takers' abilities. In general, the CAT system consists of five main components, namely [2-4]:

- calibrated item bank,
- the starting point,
- item selection rule,
- scoring, and
- stopping rule.

Given 1 and 2, repeat 3 and 4 until 5 is satisfied.

At the stage of item selection, an algorithm is needed that can estimate the ability of the examinees so that the items to be issued are adjusted to the abilities of the participants. The classic method of knowing the level of participants' ability is to the Maximum Likelihood estimator (MLE) [5,6]. The Bayesian approach used in [7,8] to estimate participants' ability.

In this research, the estimation of participants' abilities will be carried out using fuzzy logic. Fuzzy logic estimates the ability of examinees based on the difficulty level of the question, the parameters of the question and the response/answers of the examinees. The estimation results are used to determine the next question that will be issued.

2. Method

In general, Fuzzy Logic Control (FLC) has four main parts, as shown in Figure 1. The four elements have the following functions [9,10]:

- Fuzzifier functions to transform input signals that are crisp to the fuzzy set using the fuzzifier operator.
- The Knowledge Base contains a database and basic rules that define fuzzy sets of input and output areas and arrange them in the control rules.
- Decision Making is at the core of FLC, which can be human in making decisions. Fuzzy set actions are inferred using fuzzy implications and fuzzy inference mechanisms.
- Defuzzification functions to transform conclusions about fuzzy set actions into an actual variable that is crisp by using defuzzifier operators.

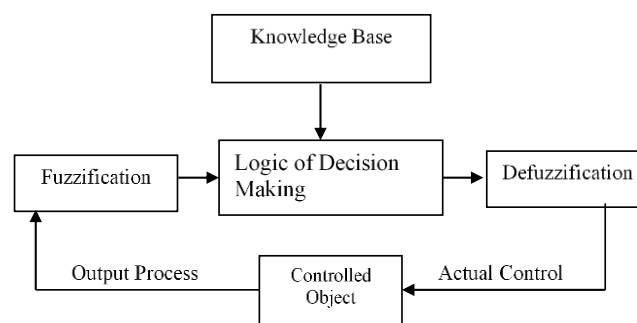


Figure 1. Fuzzy logic control.

In this research, the block diagram of the fuzzy logic system to estimate the ability of examinees, as shown in Figure 2. There are four inputs to the fuzzy system, namely:

- Item difficulty level (b_i), with three levels, namely easy, medium, and difficult.
- Question discrimination (a_i), with two levels, namely satisfactory and good.
- Probability of the examinees answer the question correctly (p_i), calculated using Equation 1 [6].

$$\rho_i(\theta) = c_i + (1 - c_i) \frac{e^{1.7a_i(\theta - b_i)}}{1 + e^{1.7a_i(\theta - b_i)}} \quad i = 1, 2, 3, \dots, n \tag{1}$$

where,

$p_i(\theta)$: the probability that the student has the ability θ to answer the item i correctly

θ : student's ability to estimate

b_i : item difficulty level

a_i : item discrimination

c_i : guessing factor

n : the number of items

e : 2.718

The parameter c is worth 0.25 because there are only four possible answers to each question (multiple choice questions). This input is divided into two categories, namely minimum and maximum.

- Participant's response or answer (r_i), if the correct answer is worth 1, whereas if the wrong answer is 0.

Input 1 and input 2 are the question parameters.

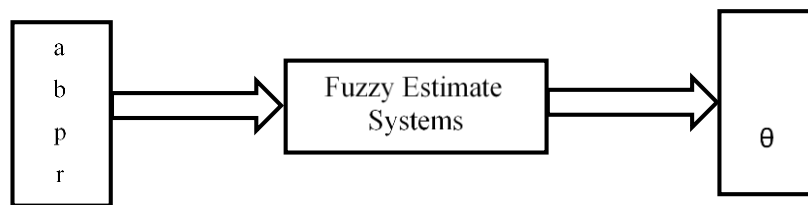


Figure 2. Block diagram of the fuzzy estimate system.

Fuzzy system output is the estimated value of the participant's ability (θ). This output is divided into five levels, namely Very Low (VL), Low (L), Average (Av), Great (G), and Excellent (Ex). These five output levels adjust to the values in the student scoring system, which is from the highest value A to the lowest value E. While the values B, C, and D are in the two values.

The membership functions of the four inputs and one output can be seen in Figure 3 to Figure 7.

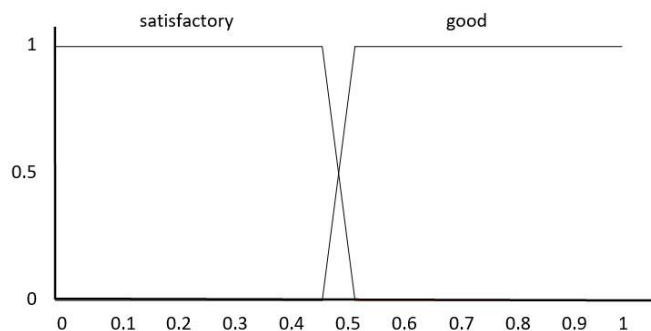


Figure 3. The membership function of the input variable “item discrimination”.

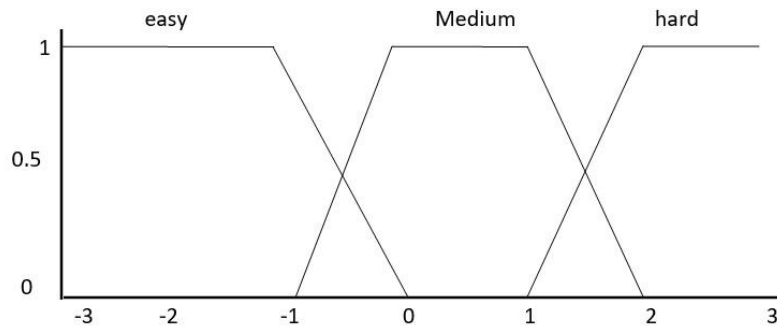


Figure 4. The membership function of the input variable “item difficulty level”.

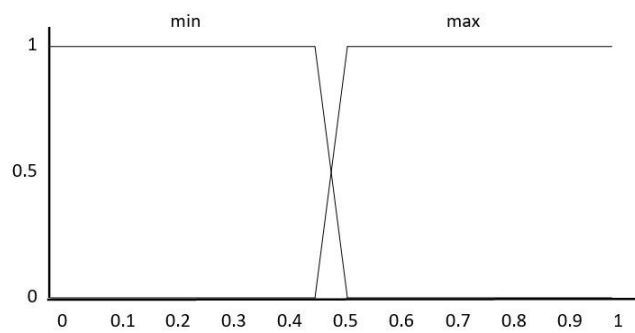


Figure 5. The membership function of the input variable “Probability”.

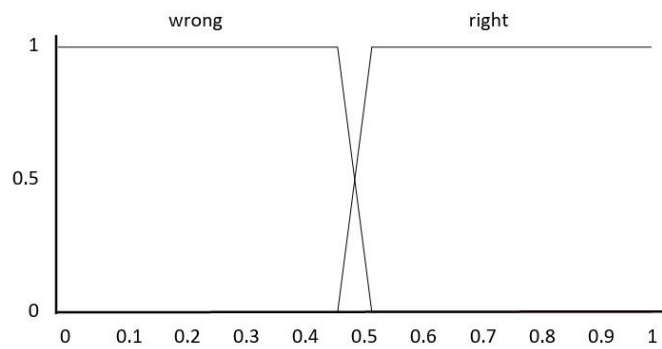


Figure 6. The membership function of the input variable “participants’ response”.

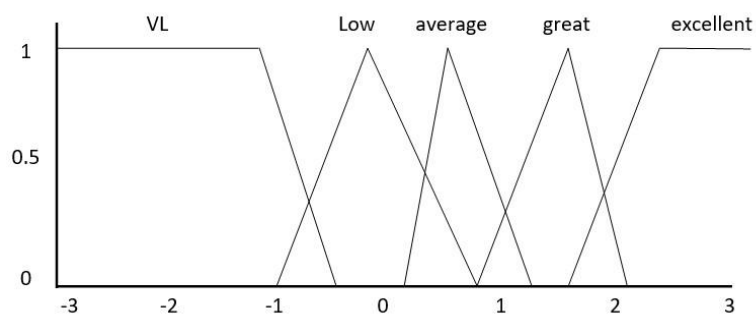


Figure 7. The membership function of output variable “participants' abilities level”.

Based on the above input-output fuzzy system, fuzzy system rules are then compiled to estimate the ability of the examinees. There are 4 inputs namely item discrimination (divided into 2 levels), item difficulty level (divided into 3 levels), probability (divided into 2 levels) and participant's response (divided into 2 levels), so that the combination of these four inputs produces 24 rules (= 2 times 3 times 2 times 3). The 24 fuzzy IF-THEN rules summarized in Table 1. The value 1 in Table 1 shows input or output at that level. The explanation of Table 1 given below.

- IF discrimination is Satisfactory AND difficulty is Easy AND probability is Minimum AND response is Wrong THEN ability is Very Low (rule 1).
- IF discrimination is Satisfactory AND difficulty is Easy AND probability is Minimum AND response is Right THEN ability is Low (rule 2).
- IF discrimination is Satisfactory AND difficulty is Medium AND probability is Minimum AND response is Right THEN ability is Average (rule 6).
- IF discrimination is Good AND difficulty is Medium AND probability is Minimum AND response is Right THEN ability is Good (rule 18).
- IF discrimination is Good AND difficulty is Hard AND probability is Maximum AND response is Right THEN ability is Excellent (rule 24).

The logic that underlies rule number 1 is that if an exam participant is given an easy question, and he/she answers incorrectly, the level of ability is very low. Other fuzzy rules are built based on relevant logic.

Fuzzy system designed using Mamdani fuzzy model with the defuzzification process using center of gravity defuzzifier.

Table 1. Fuzzy system rules.

Rule No.	Input										Output Ability (θ)				
	Discrimination (a)		Difficulty (b)			Probability (p)		Response (r)			VL	L	Av	G	Ex
	G	S	H	M	E	Max	Min	R	W						
1		1			1			1	1	1					
2		1			1			1	1		1				
3		1			1	1			1	1					
4		1			1	1		1					1		
5		1		1				1		1		1			
6		1		1				1	1				1		
7		1		1		1			1		1				
8		1		1		1		1						1	
9		1	1					1		1			1		
10		1	1					1	1					1	
11		1	1			1			1				1		
12		1	1			1		1		1					1
13	1				1			1		1		1			
14	1				1			1	1				1		
15	1				1	1			1	1					
16	1				1	1		1					1		
17	1			1				1		1			1		
18	1			1				1	1					1	
19	1			1		1			1		1				
20	1			1		1		1							1
21	1		1					1		1			1		
22	1		1					1	1						1
23	1		1			1			1				1		
24	1		1			1		1		1					1

3. Result and discussion

The design of the fuzzy system to estimate the ability of examinees, then simulated using system simulation software. For simulation purposes, we use the item bank for Linear Algebra Course. There are 177 multiple choice questions for six topics, namely vector, matrices concepts, matrices operation, determinant, inverse, and matrices applications.

We can see the results of this simulation to what extent the fuzzy system that is built can produce the expected output. As the first question, a question with a moderate level (Medium) is chosen, and the probability of correct answer = 0.5. Table 2 to Table 4 shows the different scenarios of simulation for the Vector topic.

The first scenario is if the answer continues right (see Table 2). In this case, the difficulty level of the question will increase (medium to hard level) along with the increase in the estimation of participants' abilities (θ). The test for this subject will stop at the fourth question because, at that time, the estimated ability value (θ) has not changed anymore.

Table 2. Simulation results of vector topics if the response continues right.

Stage	Level	No. of Question	a	b	p	r	c	θ	$\Delta\theta$
1	M	16	0,8	-0,4055	0,5	1	0,25	1,874	
2	H	28	0,1	1,7346	0,6294	1	0,25	2,207	0,333
3	H	29	0,1	2,9444	0,6015	1	0,25	2,4589	0,2519
4	H	30	0,1	2,9444	0,6095	1	0,25	2,4589	0

The second scenario is if the participant's answer is wrong. In Table 3 can be seen that the level of difficulty of the question will go down (from medium to easy) along with the decrease in the estimation of the ability of the examinee (θ). The test will stop at the third question because the value of (θ) no longer changes.

Table 3. Simulation results of vector topics if the response continues wrong.

Stage	Level	No. of Question	a	b	p	r	c	θ	$\Delta\theta$
1	M	16	0,8	-0,4055	0,5	0	0,25	-1,0786	
2	E	12	0,5	-1,0986	0,6282	0	0,25	-1,8853	-0,8067
3	E	8	0,3	-1,7346	0,6105	0	0,25	-1,8853	0

The third scenario is if the participants' answers are right and wrong alternately (see Table 4). This results in the estimation of the ability (θ) to change continuously, or $\Delta\theta$ will not reach zero. Likewise, the level of difficulty of the problem will be fluctuating (medium-hard-medium). In this third case, the procedure for stopping the exam is if the number of questions has reached six items.

For other topics (namely matrices concepts, matrices operation, determinant, inverse, and matrices applications), we obtain the simulation result that tends to be the same as the results on the vector topic.

Table 4. Simulation results of vector topics if response right and wrong alternately.

Stage	Level	No. of Question	a	b	p	r	c	θ	$\Delta\theta$
1	M	16	0,8	-0,4055	0,5	1	0,25	2,0819	
2	H	28	0,1	1,7346	0,6360	0	0,25	0,2873	-1,7946
3	M	21	0,8	0,4055	0,5949	1	0,25	2,4589	2,1716
4	H	29	0,1	2,9444	0,6095	0	0,25	0,6657	-1,7932
5	M	26	0,3	0,619	0,6294	1	0,25	1,5	0,8343
6	H	30	0,1	2,9444	0,5791	0	0,25	0,6657	-0,8343

4. Conclusion

It has been successfully designed a Computerized Adaptive Test (CAT) with an estimation of participants' abilities using the Mamdani Fuzzy Inference System. The simulation process was carried out for Linear Algebra Courses with six topics, namely vector, matrices concepts, matrices operation, determinant, inverse, and matrices applications. The simulation results show that for each topic, at most 6 questions are delivered, and the system has been able to know the participants' abilities.

Acknowledgments

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