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Computerized Adaptive Test based on Sugeno Fuzzy Inference System

W Ridwan*, I Wiranto and R D R Dako

Electrical Engineering Department, State University of Gorontalo, Jalan Jendral Sudirman no.6 Kota Gorontalo, Indonesia 96128

*wridwan@ung.ac.id

Abstract. Along with the development of information and communication technology, assessment of student learning outcomes is no longer carried out in the form of written examinations but instead carried out with a Computerized Adaptive Test (CAT). CAT is adaptive because it allows the items given are selected according to the ability of students. Therefore, the CAT needs a method to estimate student ability. This research aims to design a Sugeno Fuzzy Inference System (SFIS) to estimate student ability. This fuzzy system consists of twelve IF-THEN rules, with four inputs, namely the student's answer, the probability of students being able to answer correctly, the level of difficulty and discrimination of the questions. The output is an estimated ability, divided into five levels, namely very low, low, average, great, and excellent. Fuzzy system simulation is performed on linear algebra course (consist of six subject) with multiple-choice questions. The simulation results show that the SFIS can estimate the ability of students by working on a maximum of six items each subject. The estimated value of student ability obtained which is not much different from previous research (with twenty four rules). Therefore, this proposed system is more efficient.

1. Introduction

In the assessment process, one technique is a written test, which can be carried out at the end of the learning process, mid-semester, and the end of the semester. Learning outcomes assessment activities in the cognitive/knowledge domain are usually carried out with objective test techniques and descriptions of various types and carried out conventionally using paper or also called pencil and paper test. Along with the development of information and communication technology, the assessment of student learning outcomes can be done computerized (Computer Based Test/CBT). Some of the advantages of implementing CBT are that it can reduce the use of paper (following the principle of environmentally friendly), allows randomization of questions between one student and other students, reduces subjectivity and is not affected by the lecturer condition (tired, unfocused) in conducting assessments [1]. The Computerized Adaptive Test (CAT) is a form of computerized test/CBT where the items are given selected according to students' abilities [2].

The choice of items is determined by looking at the previous answers. If the answer is correct, then students are given a more challenging level of questions and vice versa. The CAT system consists of five main components, namely [3–5]:

- calibrated item bank,
- the starting point,

- item selection rule,
- scoring, and
- stopping rule.

Given 1 and 2, repeat 3 and 4 until 5 is satisfied.

The important component in the item selection stage is the algorithm for estimating the ability of the examinees. The classic method that is widely used is the Maximum Likelihood Estimator (MLE) [6,7]. Also, the Bayesian approach is used in [8,9]. Along with the development of artificial intelligence (AI), AI methods began to be used in estimating the ability of participants in the CAT system, the example is in [1,2,10] using a fuzzy system. In [1], the fuzzy system used is the Mamdani fuzzy model with 24 IF-THEN rules.

In this research, we design the method of estimating the ability of examinees uses the Sugeno fuzzy model or also known as the Takagi Sugeno Kang (TSK) model [11]. The proposed estimation system will be compared to the previous research in [1].

2. Methods

A fuzzy rule in a Sugeno fuzzy model has the form.

$$\text{if } x \text{ is } A \text{ and } y \text{ is } B \text{ then } z = f(x,y) \tag{1}$$

where A and B are fuzzy sets in the antecedent, while $z = f(x,y)$ is a crisp function in the consequent and usually $f(x,y)$ is a polynomial in the input variables x and y [11]. When f is a constant, the model called zero-order Sugeno fuzzy, in which each rule's consequent is specified by a fuzzy singleton. This research uses the zero-order Sugeno Fuzzy Model.

Input-Output fuzzy system for estimating the ability of examinees to follow the model in [1] with the following specifications:

- Input fuzzy consist of four variables, namely item difficulty level (b_i), with three levels, namely easy, medium, and difficult; question discrimination (a_i), with two levels, namely satisfactory and good; probability of the examinees answer the question correctly (p_i), with two levels, namely min and max; participant's response or answer (r_i), if the correct answer is worth 1, whereas if the wrong answer is 0.
- Fuzzy Output is the estimated value of the examinee's abilities (θ), divided into five levels, namely Very Low (VL), Low (L), Average (Av), Great (G), and Excellent (Ex).
- Fuzzy Inference System uses Sugeno Fuzzy Model, in contrast to [1], which uses Mamdani Fuzzy Inference Systems.
- The membership functions (μ) of the four inputs can be seen in Figure 1 to Figure 4 [1].

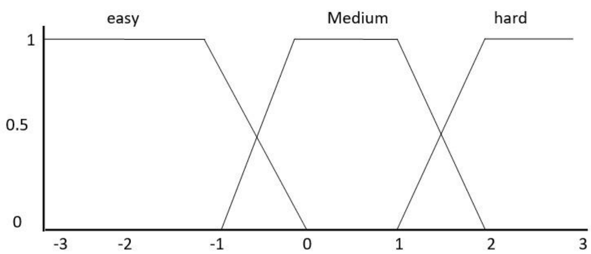

Figure 1. Membership function of item difficulty level.

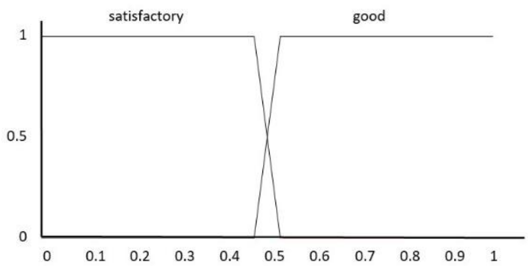

Figure 2. Membership function of item discrimination.

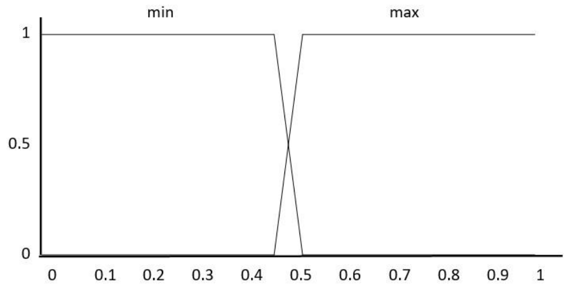

Figure 3. Membership function of probability.

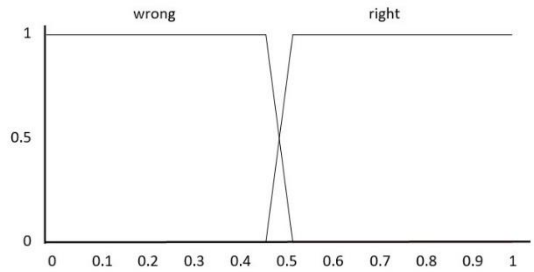

Figure 4. Membership function of participants' response.

Based on the fuzzy input-output above, fuzzy system rules are then arranged to estimate the ability of the examinees. There are 12 IF-THEN Fuzzy Rules as follows:

- IF *discrimination* is *Satisfactory* AND *difficulty* is *Easy* AND *probability* is *Maximum* AND *response* is *Wrong* THEN *ability* is *Very Low*
- IF *discrimination* is *Satisfactory* AND *difficulty* is *Easy* AND *probability* is *Maximum* AND *response* is *Right* THEN *ability* is *Average*
- IF *discrimination* is *Satisfactory* AND *difficulty* is *Medium* AND *probability* is *Maximum* AND *response* is *Wrong* THEN *ability* is *Low*
- IF *discrimination* is *Satisfactory* AND *difficulty* is *Medium* AND *probability* is *Maximum* AND *response* is *Right* THEN *ability* is *Excellent*
- IF *discrimination* is *Satisfactory* AND *difficulty* is *Hard* AND *probability* is *Maximum* AND *response* is *Wrong* THEN *ability* is *Average*
- IF *discrimination* is *Satisfactory* AND *difficulty* is *Hard* AND *probability* is *Maximum* AND *response* is *Right* THEN *ability* is *Excellent*
- IF *discrimination* is *Good* AND *difficulty* is *Easy* AND *probability* is *Maximum* AND *response* is *Wrong* THEN *ability* is *Very Low*
- IF *discrimination* is *Good* AND *difficulty* is *Easy* AND *probability* is *Maximum* AND *response* is *Right* THEN *ability* is *Average*
- IF *discrimination* is *Good* AND *difficulty* is *Medium* AND *probability* is *Maximum* AND *response* is *Wrong* THEN *ability* is *Low*
- IF *discrimination* is *Good* AND *difficulty* is *Medium* AND *probability* is *Maximum* AND *response* is *Right* THEN *ability* is *Excellent*
- IF *discrimination* is *Good* AND *difficulty* is *Hard* AND *probability* is *Maximum* AND *response* is *Wrong* THEN *ability* is *Average*
- IF *discrimination* is *Good* AND *difficulty* is *Hard* AND *probability* is *Maximum* AND *response* is *Right* THEN *ability* is *Excellent*

The constants for Sugeno Fuzzy Inference Systems (SFIS) output are Very Low = -1.8597, Low = -0.0667, Average = 0.6667, Great = 1.5, and Excellent = 2.4697.

3. Results and discussion

The simulation is done by taking bank item data for Linear Algebra Course [1]. As an example of calculation, suppose discrimination (a) = 0.8, difficulty (b) = -0.40547, probability (p) = 0.5, and response (r) = 1. Then based on the membership function each input is obtained,

$$\begin{gathered}\mu_{a(\text{satisfactory})} = 0,\ \mu_{a(\text{good})} = 1,\ \mu_{b(\text{easy})} = 0.40547,\ \mu_{b(\text{medium})} = 0.59453, \\ \mu_{p(\text{min})} = 0,\ \mu_{p(\text{max})} = 1,\ \mu_{r(\text{wrong})} = 0, \text{ and } \mu_{r(\text{right})} = 1\end{gathered} \tag{2}$$

Then the values in (2) are included in the fuzzy rules. Fuzzy rules use AND logic, so the minimum value is taken at the antecedent. Based on this, the fuzzy rules that apply only rule number two and number four (other rules are zero), so the ability (θ) is obtained,

$$\theta = \min(1,0.40547,1,1) \times 0.6667 + \min(1,0.59453,1,1) \times 1.5 = 1.7386 \quad (3)$$

The estimated value of the examinees' abilities is 1.7386 so that the next item is chosen which difficulty level is close to 1.7386.

Table 1 shows a simulation where the examinees answered continues right. Table 2 shows a simulation if the participant's response continues wrong, and Table 3 shows the participant's answer right and wrong alternately. The test will stop if the estimated value of the student's ability does not change from the previous item. If the examinee's response right and wrong alternately, the exam will stop if the number of questions has reached six items (Table 3). The three tables also show the comparison of the estimated ability of participants based on Mamdani Fuzzy Inference Systems [2]. Table 1 to Table 3 shows the difference in the results of the estimation of the ability of examinees with the Fuzzy Mamdani Model (θ_M), and Fuzzy Sugeno (θ_S) did not differ much, even in some cases there was no difference. It shows that the Fuzzy Sugeno method used in this research is better because it only applies 12 fuzzy rules compared to [1], which uses 24 fuzzy rules.

Table 1. Simulation results if the response continues right.

Stage	Diff. level	No. of question	*a*	*b*	*p*	*r*	θ_M	θ_S	*Selisih* $\lvert(\theta_M - \theta_S)\rvert$
1	M	16	0.8	-0.4055	0.5	1	1.8740	1.7386	0.1354
2	H	28	0.1	1.7346	0.6294	1	2.2070	2.2044	0.0026
3	H	29	0.1	2.9444	0.6015	1	2.4589	2.4589	0
4	H	30	0.1	2.9444	0.6095	1	2.4589	2.4589	0

Table 2. Simulation results if the response continues wrong.

Stage	Diff. level	No. of question	*a*	*b*	*p*	*r*	θ_M	θ_S	*Selisih* $\lvert(\theta_M - \theta_S)\rvert$
1	M	16	0.8	-0.4055	0.5	0	-1.0786	-0.804	0.2746
2	E	12	0.5	-1.0986	0.6282	0	-1.8853	-1.6076	0.2777
3	E	8	0.3	-1.7346	0.6105	0	-1.8853	-1.8853	0
4	E	7	0,1	-1,7346	0,620197	0	-1.8853	-1.8853	0

Table 3. Simulation results if response right and wrong alternately.

Stage	Diff. level	No. of question	*a*	*b*	*p*	*r*	θ_M	θ_S	*Selisih* $\lvert(\theta_M - \theta_S)\rvert$
1	M	16	0,8	-0,4055	0,5	1	2,0819	1,7318	0.3501
2	H	28	0,1	1,7346	0,6360	0	0,2873	0,4714	0.1841
3	M	21	0,8	0,4055	0,5949	1	2,4589	2,4589	0
4	H	29	0,1	2,9444	0,6095	0	0,6657	0,6657	0
5	M	26	0,3	0,619	0,6294	1	1,5	1,5	0
6	H	30	0,1	2,9444	0,5791	0	0,6657	0,4714	0.1943

4. Conclusion

This research has been successfully designing Sugeno Fuzzy Inference Systems (SFIS) to estimate the ability of examinees on the Computerized Adaptive Test (CAT) system. This SFIS consists of four inputs, one output, and twelve fuzzy rules. In the simulation, the estimated value of fuzzy ability is obtained which is not much different from previous research (with twenty four rules). Therefore, this proposed estimation system is more efficient.

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