Networked Control Synthesis Using Time Delay Approach: State Feedback Case

Wrastawa Ridwan[1], Bambang Riyanto Trilaksono[2]

[1]Department of Electrical Engineering, Gorontalo State University, Gorontalo, Indonesia
Jl. Jendral Sudirman no.6 Kota Gorontalo, Indonesia 96128
[2]School of Electrical Engineering & Informatics, Bandung Institute of Technology,
Bandung, Indonesia, Jl. Ganeca no. 10 Bandung, Indonesia 40132
r1space@yahoo.com, briyanto@lskk.ee.itb.ac.id

Abstract: Networked Control Systems (NCS) is a control system in which the sensors, controllers, and the actuators are connected via communication network. The use of communication network will lead to intermittent losses or delays of the information and may decrease the performance and cause instability. This paper presents the design of H_∞ state feedback controller for NCS. The NCS is modelled as a time delay system. Two network features are considered: signal transmission delay and data packet dropout. Our objective is focused on the design of state feedback controller which guarantee asymptotic stability of the closed-loop systems. The proposed methods are given in the terms of Linear Matrix Inequality (LMI). Finally, we consider an unstable system for numerical example. It is shown that the state feedback controller proposed here make the closed-loop system stable with or without input disturbance.

Keywords: Networked Control Systems (NCS), Signal Transmission Delay, Data Packet Dropout, State Feedback Controller

1. Introduction

In modern control systems, physical plant, controller, sensors and actuator are difficult to be located at the same place, and hence these components need to be connected over network media. When feedback control system is closed via a communication channel, then the control system is classified as a Networked Control System (NCS) [1,2]. NCS has been attracted much attention due to significant advantages, such as reduced installation and maintenance cost, increased system agility, and so on [3]. There have many researcher to conduct research on the topic of NCS. To mention a few, [4] addressed the problem of quantized feedback control. Result on state feedback control could be seen in [5,6]. Stability analysis, and stabilization of NCS are investigated in [2, 3] and the references therein. There are some parameters that arise when using communication network in NCS such as packet dropouts and signal transmission delay. The presence of these parameters can degrade the performance of the system and could lead to instability.

When information or energy is physically transmitted from one place to another, there is a delay associated with the transmission [7]. It is well known that the presence of time-delay is a source of instability [8]. Xia et al. [9] presents some basic theories of stability and synthesis of systems with time-delay, in the form of $\dot{x}(t) = Ax(t) + A_d x(t-\tau(t))$, where $\tau(t)$ represents time-varying delay. Wu et al. [10] presents a method referred to as the free-weighting-matrix (FWM) approach for the stability analysis and control synthesis of various classes of time-delay systems. In [11], a new model for time delay systems is proposed, that is $\dot{x}(t) = Ax(t) + A_d x(t-\tau_1(t)-\tau_2(t))$. The new model is motivated by practical situation in Networked Control Systems (NCSs), where $\tau_1(t)$ is the time-delay from sensor to the controller and $\tau_2(t)$ is the time-delay from controller to the actuator. Based on such a system

Received: April 10th, 2011. Accepted: December 20th, 2011

representation, [11] derived the stability condition. Gao et al. [5] presented a new stability condition and investigated the problem of H_∞ performance analysis. Dey et al. [12] constructed a new Lyapunov-Krasovskii functional in obtaining the stability condition for such system, and provided less conservative delay upper bound, as compared to the conditions in [5,11].
Motivated by [12], in this paper we continue the preliminary result derived in [12] to the NCS problem. Our objective is focused on the design of H_∞ state feedback controller for NCS. It is well known in systems and control community that H_∞-norm constraint can be used to provide a prespecified disturbance attenuation level, and alternatively to analyze robust stability of dynamical system.

This paper improve the previous paper appeared in [16], that is reduce the number of variables in Theorem 1. In addition, analysis and discussion have been improved by exploring the effect of the parameters change.

Notation. The notation $X > 0$ denotes a symmetric positive definite, asterisk (*) represents the elements of symmetric term in the symmetric block matrix. The superscripts "*T*" and "*-1*" represent the transpose and inverse matrix, respectively. $L_2[0,\infty)$ is the space of square integrable functions on [0,∞).

Before proceeding further, we give the following lemmas.
Lemma 1. Schur Complement. Schur's formula says that the following statements are equivalent [13,14]:

i. $$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0$$

ii. $$\Phi_{22} < 0$$
$$\Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{12}^T < 0$$

Lemma 2. Consider the following system with two additive time-varying delays in the state [15]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x\big(t - \tau_1(t) - \tau_2(t)\big) + Ew(t), \\ y(t) &= Cx(t) + C_d x\big(t - \tau_1(t) - \tau_2(t)\big) + Fw(t), \\ x(t) &= \phi(t), \qquad t \in [-\bar{\tau}, 0] \end{aligned} \tag{1}$$

Assumming that

$$\tau_1(t) \equiv \bar{\tau}_1 < \infty,\ 0 \le \tau_2(t) \le \bar{\tau}_2 < \infty \tag{2}$$

System (1) satisfying (2) either
(i). Asymptotically stable with $w = 0$, or
(ii). Stable with H_∞ disturbance attenuation level γ ($w \neq 0$)

if there exist matrices $P = P^T > 0$, $Q_2 = Q_2^T > 0$, $R_1 = R_1^T > 0$, $R_2 = R_2^T > 0$, $R_3 = R_3^T > 0$ and G_i, L_i, M_i, N_i, $i = 1, ..., 4$ are free matrices satisfying,

$$\begin{bmatrix} \Psi_1 + \Psi_2 + \Psi_2^T + \Psi_3 + \Psi_4 + \Psi_6^T\Psi_6 & \Psi_5 \\ * & \Xi_5 \end{bmatrix} < 0 \tag{3}$$

where,

$$\Psi_1 = \begin{bmatrix} Q_2 & 0 & 0 & P & 0 \\ * & -Q_2 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & \bar{\tau}R_1 + \bar{\tau}_1 R_2 + \bar{\tau}_2 R_3 & 0 \\ * & * & * & * & 0 \end{bmatrix}, \Psi_5 = \begin{bmatrix} L_1 & M_1 & N_1 \\ L_2 & M_2 & N_2 \\ L_3 & M_3 & N_3 \\ L_4 & M_4 & N_4 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_2 = \Psi_{21}\Psi_{22},\ \Psi_{21} = \begin{bmatrix} A & 0 & A_d & -I & E \end{bmatrix}^T,\ \Psi_{22} = \begin{bmatrix} G_1^T & G_2^T & G_3^T & G_4^T & 0 \end{bmatrix}$$

$$\Psi_3 = \begin{bmatrix} L_1 + L_1^T + M_1 + M_1^T & L_2^T - M_1 + M_2^T + N_1 & -L_1 + L_3^T + M_3^T - N_1 & L_4^T + M_4^T & 0 \\ * & -M_2 - M_2^T + N_2 + N_2^T & -L_2 - M_3^T - N_2 + N_3^T & -M_4^T + N_4^T & 0 \\ * & * & -L_3 - L_3^T - N_3 - N_3^T & -L_4^T - N_4^T & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$\Psi_4 = \text{diag}\left\{0,0,0,0,-\gamma^2 I\right\}, \Psi_6 = \begin{bmatrix} C & 0 & C_d & 0 & F \end{bmatrix}, \ \Xi_5 = \text{diag}\left\{-\bar{\tau}^{-1}R_1, -\bar{\tau}_1^{-1}R_2, -\bar{\tau}_2^{-1}R_3\right\}$$

2. System Description

In this section, we model the NCS with consideration of the effect of network-induced delay and data packet dropout. Then we design the state feedback controller. The model of NCS follow the model described in [2]. We consider an NCS shown in Figure 1. Suppose the physical plant is given by the following system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \\ y(t) &= Cx(t) + Du(t) + Fw(t) \end{aligned} \qquad (4)$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^p$ *is the control input,* $y(t) \in \Re^q$ is the output vector and $w(t) \in \Re^l$ is the disturbance input which belongs to $L_2[0,\infty)$; A, B, E, C, D, and F are known system matrices with appropriate dimension.

It is assumed that sensor and sampler are clock-driven, while the controller, ZOH and actuator are event-driven. The sampling period is assumed to be h (positive real constant) and the sampling instant are denoted as s_k, k = 1, 2, ... The measurement of state variable is $\bar{x}$ and then transmitted with a single packet.

At the sampling instant s_k, we have,

$$\bar{x}(s_k) = \begin{bmatrix} x_1(s_k) & x_2(s_k) & \ldots & x_n(s_k) \end{bmatrix}^T \qquad (5)$$

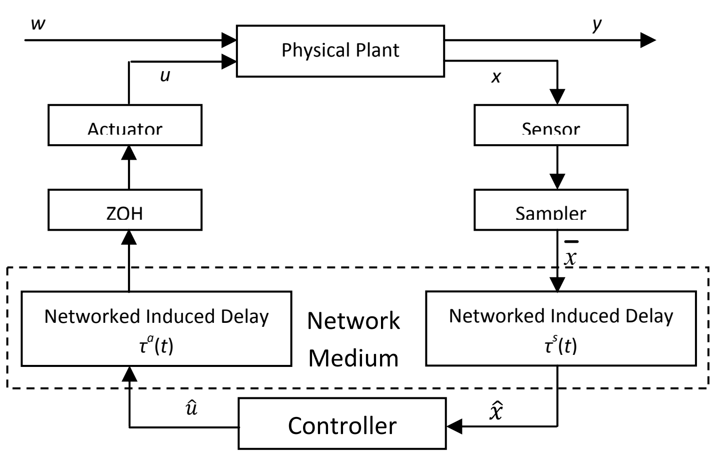

Figure 1. Networked control system

Transmitted signal from the sampler to the controller experienced signal transmission delays τ_k^s that is

$$\hat{x}(s_k) = \bar{x}\left(s_k - \tau_k^s\right)$$

where τ_k^s is the network delay from the sampler to the controller at instant *k*.

Control signal from the controller is

$$\hat{u}(s_k) = K\hat{x}(s_k) = Kx(s_k - \tau_k^s)$$

Then, transmitted signal from the controller to the ZOH experienced signal transmission delays that is

$$u(s_k) = \hat{u}(s_k - \tau_k^a)$$

where τ_k^a is the network delay from the controller to the ZOH at instant *k*.

Overall, transmitted signal from the sampler to ZOH experienced signal transmission delays η_k, that is

$$\eta_k = \tau_k^s + \tau_k^a \tag{6}$$

So that, the control signal is

$$u(s_k) = Kx(s_k - \tau_k^s - \tau_k^a) = Kx(s_k - \eta_k)$$

It is assumed that there are no delay between the sensor and the sampler and between the ZOH and the actuator.

We denote the updating instants of the ZOH as t_k, $k = 1, 2, \ldots$. The state feedback controller takes the following form:

$$u(t_k) = Kx(t_k - \eta_k) \tag{7}$$

where *K* is the state feedback gain.

Considering the behaviour of the ZOH, the control input is

$$u(t) = Kx(t_k - \eta_k), \quad t_k \le t < t_{k+1} \tag{8}$$

From (4) and (8), we obtain the following closed-loop system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BKx\left(t_k - \eta_k\right) + Ew(t), \\ y(t) &= Cx(t) + DKx\left(t_k - \eta_k\right) + Fw(t), \\ t_k &\le t \le t_{k+1} \end{aligned} \tag{9}$$

Network-induced delay always exists when the data transmits through a network and is non-differentiable interval time-varying delay. So a natural assumption on η_k can be made as [4,5]

$$0 < \eta_m \le \eta_k \le \eta_M \tag{10}$$

where η_m and η_M denote the minimum and maximum delay bounds, respectively.

The effect of data packet dropouts in the communication channel can be described as the ZOH is not updated during the time interval of this event, which is referred to vacant sampling. Hence, the effect of one packet dropout in the transmission is just a case that one sampling period delay is induced in the updating interval of the ZOH [4, 5].

$$t_{k+1} - t_k = (\delta_{k+1} + 1)h + \eta_{k+1} - \eta_k \tag{11}$$

where δ_{k+1} is the number of accumulated packet dropouts in the period.

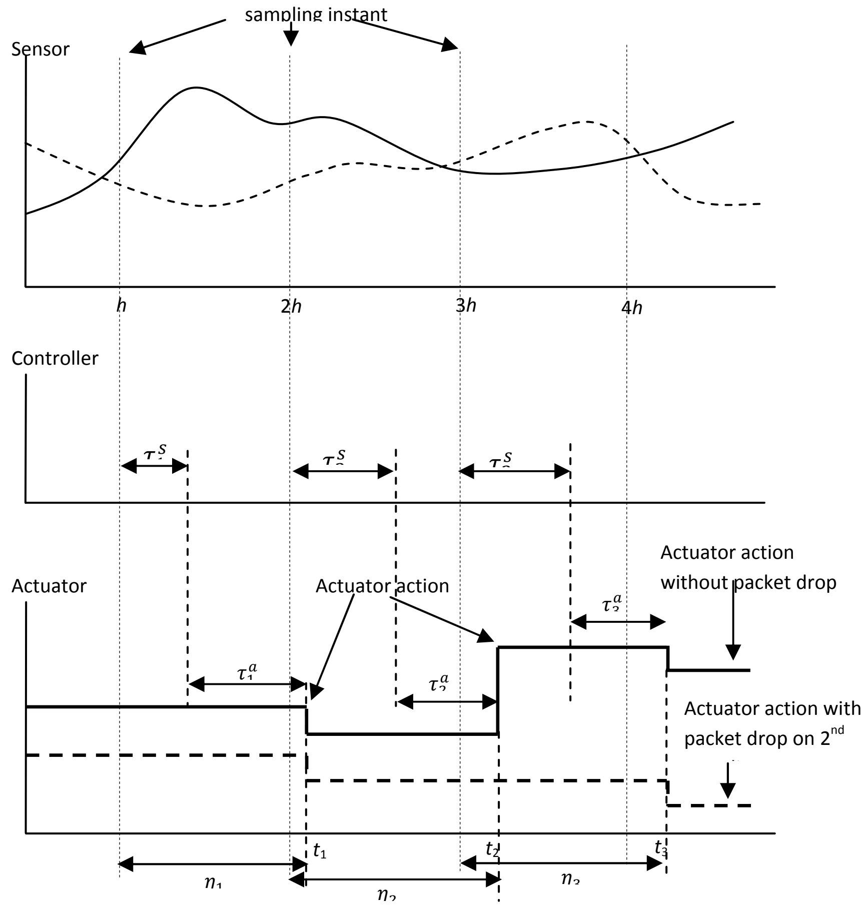

Figure 2. Timing diagram

In a similar way as in [5], let us represent $t_k - \eta_k$ in (9) as

$$t_k - \eta_k = t - \eta_m - \eta(t), \tag{12}$$

where $\eta(t) = t - t_k + \eta_k - \eta_m$, $t_k \le t < t_{k+1}$. It is obviously that

$$0 \le \eta(t) < t_{k+1} - t_k + (\eta_k - \eta_m) \tag{13}$$

By substituting (11) into (13), we have

$$0 \le \eta(t) < (\delta_{k+1} + 1)h + \eta_{k+1} - \eta_k + (\eta_k - \eta_m)$$

$$0 \le \eta(t) \le \kappa \tag{14}$$

where $\kappa = (\bar{\delta} + 1)h + \eta_M - \eta_m$, $\bar{\delta}$ denotes the maximum number of packet dropouts. We illustrate the timing diagram of NCS described above in Figure 2.

By substituting (12) into (9) we have

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BKx(t - \eta_m - \eta(t)) + Ew(t), \\ y(t) &= Cx(t) + DKx(t - \eta_m - \eta(t)) + Fw(t), \end{aligned} \tag{15}$$

3. Controller Design

In this section, we solve the problem of H_∞ state feedback controller design for NCS described in previous section. The problem we address here stated as follows: Design a state feedback controller in the form of (8) such that the closed-loop system (15) is asymptotically stable with H_∞ disturbance attenuation level γ.

The main result of the present paper stated in the following theorem.

Theorem 1. Consider the NCS in Figure1. For given scalars ρ_i, (i = 2, 3, 4), η_m, η_M, h, and $\bar{\delta}$. There exists a state feedback controller in the form of (8) such that the closed-loop system in (15) either,

(i). asymptotically stable with $w = 0$, or

(ii). asymptotically stable with H_∞ disturbance attenuation level γ ($w \neq 0$)

if there exist matrices $\bar{P} = \bar{P}^T > 0$, $\bar{Q}_2 = \bar{Q}_2^T > 0$, $\bar{R}_i = \bar{R}_i^T > 0$ (i = 1, 2, 3), non-singular matrix X, and $\bar{L}_i$, $\bar{M}_i$, $\bar{N}_i$ (i = 1,..., 4) are free matrices satisfying,

$$\begin{bmatrix} \Omega_1 + \Omega_2 + \Psi_4 & \Omega_5 & \Omega_3 \\ * & \Omega_4 & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{16}$$

where

$$\Omega_1 = \begin{bmatrix} \bar{Q}_2 + AX^T + XA^T & \rho_2 XA^T & B\bar{K} + \rho_3 XA^T & \Omega_{114} & E \\ * & -\bar{Q}_2 & \rho_2 B\bar{K} & -\rho_2 X^T & \rho_2 E \\ * & * & \rho_3 B\bar{K} + \rho_3 \bar{K}^T B^T & \Omega_{134} & \rho_3 E \\ * & * & * & \Omega_{144} & \rho_4 E \\ * & * & * & * & 0 \end{bmatrix},$$

$$\Omega_{114} = \bar{P} - X^T + \rho_3 XA^T, \ \Omega_{134} = -\rho_3 X^T + \rho_4 \bar{K}^T B^T$$

$$\Omega_{144} = v\bar{R}_1 + \eta_m \bar{R}_2 + \kappa \bar{R}_3 - \rho_4 X^T - \rho_4 X,$$

$$\Omega_2 = \begin{bmatrix} \bar{L}_1 + \bar{L}_1^T + \bar{M}_1 + \bar{M}_1^T & \bar{L}_2^T - \bar{M}_1 + \bar{M}_2^T + \bar{N}_1 & -\bar{L}_1 + \bar{L}_3^T + \bar{M}_3^T - \bar{N}_1 & \bar{L}_4^T + \bar{M}_4^T & 0 \\ * & -\bar{M}_2 - \bar{M}_2^T + \bar{N}_2 + \bar{N}_2^T & -\bar{L}_2 - \bar{M}_3^T - \bar{N}_2 + \bar{N}_3^T & -M_4^T + \bar{N}_4^T & 0 \\ * & * & -\bar{L}_3 - \bar{L}_3^T - \bar{N}_3 - \bar{N}_3^T & -\bar{L}_4^T - \bar{N}_4^T & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$\Omega_5 = \begin{bmatrix} \bar{L}_1 & \bar{M}_1 & \bar{N}_1 \\ \bar{L}_2 & \bar{M}_2 & \bar{N}_2 \\ \bar{L}_3 & \bar{M}_3 & \bar{N}_3 \\ \bar{L}_4 & \bar{M}_4 & \bar{N}_4 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_4 = \text{diag}\{0,0,0,0,-\gamma^2 I\}, \Omega_4 = \text{diag}\{-v^{-1}\bar{R}_1, -\eta_m^{-1}\bar{R}_2, -\kappa^{-1}\bar{R}_3\},$$

$$\Omega_3 = \begin{bmatrix} CX^T & 0 & D\bar{K} & 0 & F \end{bmatrix}^T, \ v = \eta_m + \kappa \text{ and } \kappa = (\bar{\delta} + 1)h + \eta_M - \eta_m.$$

If condition in (16) feasible, a desired controller gain matrix is given by

$$K = \bar{K}\left(X^T\right)^{-1} \tag{17}$$

Proof. By comparing system (15) and system (1) satisfying (2), according to Lemma 2, we know that system (15) is asymptotically stable with H_∞ disturbance attenuation level γ if there exist matrices $P = P^T > 0$, $Q_2 = Q_2^T > 0$, $R_i = R_i^T > 0$ (i = 1, 2, 3), and G_i, L_i, M_i, N_i, (i = 1,..., 4) are free matrices satisfying,

$$\begin{bmatrix} \phi_4 + \phi_2 + \phi_2^T + \Psi_3 + \Psi_4 + \phi_3\phi_3^T & \Psi_5 \\ * & \phi_1 \end{bmatrix} < 0 \tag{18}$$

where

$$\phi_4 = \begin{bmatrix} Q_2 & 0 & 0 & P & 0 \\ * & -Q_2 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & vR_1 + \eta_m R_2 + \kappa R_3 & 0 \\ * & * & * & * & 0 \end{bmatrix}$$

$v = \eta_m + \kappa$, $\kappa = (\bar{\delta} + 1)h + \eta_M - \eta_m$,

$\phi_2 = \phi_{21}\Psi_{22}$, $\phi_{21} = \begin{bmatrix} A & 0 & BK & -I & E \end{bmatrix}^T$, $\phi_3 = \begin{bmatrix} C & 0 & DK & 0 & F \end{bmatrix}^T$,

$\phi_1 = \text{diag}\{-v^{-1}R_1, -\eta_m^{-1}R_2, -\kappa^{-1}R_3\}$, and Ψ_{22}, Ψ_3, Ψ_4, Ψ_5 are given in (3).

By Lemma 1 (Schur complement), (18) is equivalent to

$$\begin{bmatrix} \phi_4 + \phi_2 + \phi_2^T + \Psi_3 + \Psi_4 & \Psi_5 & \phi_3 \\ * & \phi_1 & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{19}$$

Define $G_1 = G_0$ and $G_i = \rho_i G_0$ (i = 2, 3, 4; $\rho_i \neq 0$), (19) become

$$\begin{bmatrix} \phi_1 + \bar{\phi}_2 + \bar{\phi}_2^T + \Psi_3 + \Psi_4 & \Psi_5 & \phi_3 \\ * & \phi_1 & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{20}$$

where

$\bar{\phi}_2 = \phi_{21}\bar{\Psi}_{22}$, $\bar{\Psi}_{22} = \begin{bmatrix} G_0^T & \rho_2 G_0^T & \rho_3 G_0^T & \rho_3 G_0^T & 0 \end{bmatrix}$, ϕ_{21} is given in (18).

Define $J = \text{diag}\{J_1, J_2, I\}$, $J_1 = \text{diag}\{X, X, X, X, I\}$, $J_2 = \text{diag}\{X, X, X\}$, $X = G_0^{-1}$.

Applying congruence transformation [13] to (20) by J, we get

$$\begin{bmatrix} J_1(\phi_4 + \bar{\phi}_2 + \bar{\phi}_2^T + \Psi_3 + \Psi_4)J_1^T & J_1\Psi_5 J_2^T & J_1\ \phi_3 \\ * & J_2\ \phi_1\ J_2^T & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{21}$$

Simplifying (21) together with the change of variables defined by,

$\bar{L}_i = XL_iX^T$, $\bar{M}_i = XM_iX^T$, $\bar{N}_i = XN_iX^T$ (i = 1, ..., 4),

$\bar{R}_i = XR_iX^T$ (i = 1, 2, 3), $\bar{Q}_2 = XQ_2X^T$, $\bar{K} = KX^T$, $\bar{P} = XPX^T$