Forecasting COVID-19 Epidemic in Spain and Italy Using A Generalized Richards Model with Quantified Uncertainty

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Abstract

The Richards model and its generalized version are deterministic models that are often implemented to fit and forecast the cumulative number of infective cases in an epidemic outbreak. In this paper we employ a generalized Richards model to predict the cumulative number of COVID-19 cases in Spain and Italy, based on available epidemiological data. To quantify uncertainty in the parameter estimation, we use a parametric bootstrapping approach to construct a 95% confidence interval estimation for the parameter model. Here we assume that the time series data follow a Poisson distribution. It is found that the 95% confidence interval of each parameter becomes narrow with the increasing number of data. All in all, the model predicts daily new cases of COVID-19 reasonably well during calibration periods. However, the model fails to produce good forecasts when the amount of data used for parameter estimations is not sufficient. Based on our parameter estimates, it is found that the early stages of COVID-19 epidemic, both in Spain and in Italy, followed an almost exponentially growth. The epidemic peak in Spain and Italy is respectively on 2 April 2020 and 28 March 2020. The final sizes of cumulative number of COVID-19 cases in Spain and Italy are forecasted to be at 293220 and 237010, respectively.

Keywords: Generalized Richards Models, Uncertainty, Boostraap, COVID-19.

2010 MSC classification number: 92B05, 92D25, 92D30, 97Mxx.

1. Introduction

Several cases of severe pneumonia without any known cause were reported by the Wuhan Municipal Health Committee in late 2019 [1], [2], [3]. The respiratory infections were epidemiologically linked to the Wuhan seafood market, Province of Hubei, China [4]. One week later, the unexplained pneumonia was identified as an impact of a new type of coronavirus after laboratory tests [1]. The pathogenic virus causes intestinal and respiratory infections in animal and humans [5]. On 11 March 2020, the World Health Organization (WHO) officially announced the pneumonia infection caused by a novel betacoronavirus known as *2019-nCoV* as a pandemic and named it Coronavirus Disease 2019 (*COVID-19*) [6], [7], [8]. Only in a few months, the outbreak of COVID-19 is ongoing worldwide and affecting 213 countries and territories around the world, and two international conveyances [9]. Inevitably, all countries are faced with the obstruction of economic growth which leads to a global economic crisis.

Europe as one of the driving forces of the world economy was also affected by this outbreak. Among them, Italy and Spain are considerably interesting because of their demographic characteristics, i.e., they are two of the most visited countries due to their cultural [10] as well as natural [11] tourism sectors. Certainly this means that there are high rates of travel intensity which of course gives a big impact to the spread of COVID-19 in those countries. Unfortunately, Spain and Italy are also included in the list of European countries with the highest cumulative deaths due to COVID-19 [9], although both of them immediately applied "lockdown" [12], [13]. Studying the dynamics of COVID-19 cases in Italy and Spain is therefore crucial and to be implemented immediately.

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Mathematical studies on the dynamics of COVID-19 in Italy and Spain mostly focused on forecasting the cumulative number of cases, such as Sahin and Sahin [14], Fanelli and Piazza [15], Chintalapudi et al. [16], and Ahmar et al. [17]; some others are interested in estimating the basic reproduction number, such as Zhuang [18], Chintalapudi et al. [19], D'Arienzo et al. [20], Hyafil et al. [21] and Susanto et al. [22].

It is interesting to note that the time-series of cases in many countries that have succeeded in suppressing the rate of COVID-19 transmission are in an S-shaped curve which could be described well by logistic-type models. Therefore, utilizing the growth model for predicting the cumulative number of cases is a convincing choice. For example, some researchers used the logistic growth model to study the number of COVID-19 cases in China [23], India [24], and several countries [25]. By considering that the epidemic may have a different growth-decelerating pattern and as such, the symmetric S-shaped curve may deviate, in this paper we apply a more general type of logistics model, namely a generalized Richards model (GRM).

The GRM will be implemented to fit the time series data of COVID-19 cases, specifically in Spain and Italy. Our results show that for both countries, the COVID-19 early growth is indeed sub-exponential. Furthermore, we also perform parameter uncertainty analysis by assuming that the time series data follow a Poisson distribution, from which we obtain that the fit depends on the length of reported data cases. By taking optimal parameter values, the GRM can be used to give good forecasts of daily new cases within a reasonable time period.

Our report is presented using the following outlines. We first present the data which we use to fit the mathematical model in Section 2. The model, parameter estimation method which includes uncertainty, model calibrations and forecasting are given Section 3. Results and discussion are presented in Section 4. Finally, we provide some concluding remarks in the last section.

2. Data

Data used in this work are daily cumulative cases and new daily cases of COVID-19 in Spain and Italy. There are various data sources on the internet. For our study we have chosen to use data from [9] that reports daily cumulative cases and new daily cases for all countries in the world. Dataset was taken on 21 June 2020, which includes cases reported until 20 June 2020. The reported data of cumulative number of cases in the early days are usually constant. In other words, new cases of COVID-19 were not reported regularly initially. By considering this fact, we only use reported data from 25 February - 20 June 2020 for cases in Spain and from 20 February - 20 June 2020 for cases in Italy. As mentioned in [9], the Spanish government decreased the number of total cases by 372. In consideration that the total daily new cases cannot be negative, we adjusted the total cases on 25 May 2020 by taking an average between the total cases on 24 May 2020 and on 26 May 2020. Bar charts of cumulative number of COVID-19 cases in Spain and in Italy are shown in Figure 1. The figures show that the number of cases in Spain and in Italy is saturated at a certain value which is respectively about 3×10^5 and 2.4×10^5.

3. Research Methodology

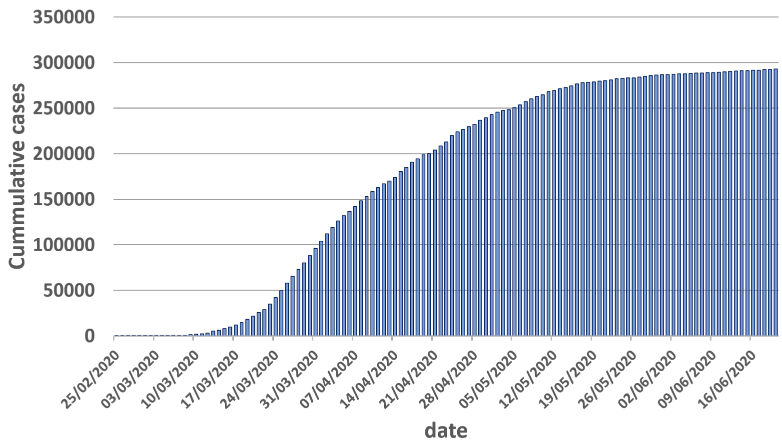
3.1. Mathematical Model

The Richards model (RM)

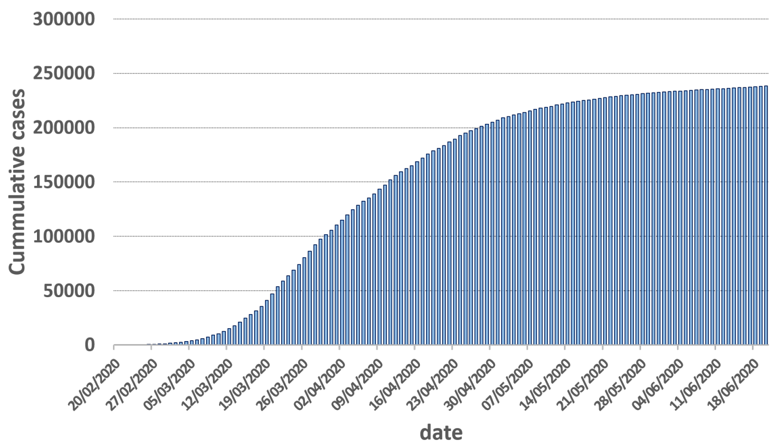
$$\frac{dC(t)}{dt} = rC(t)\left(1 - \left(\frac{C(t)}{K}\right)^a\right), \tag{1}$$

is considered as a generalization of logistic models and was initially developed to study the growth of ecological populations. Recently, this model has been applied to study epidemic outbreak [26]. It has been successfully applied to fit real time data and predict epidemic outbreak of SARS [27], [28], influenza H1N1 [29], Zika [30], and COVID-19 [31], [32], [33]. In Equation (1), $C(t)$ denotes the cumulative number of cases at time t. The model has three parameters, namely the intrinsic growth rate (r), the carrying capacity or the final epidemic size (K) and the parameter accounting for the deviation of the symmetric S-shaped of the logistic equation (a). Recently, Viboud et al. [34] and Chowell [35] introduced a 'deceleration of growth' parameter (p) to extend the RM to get the following GRM

$$\frac{dC(t)}{dt} = rC(t)^p\left(1 - \left(\frac{C(t)}{K}\right)^a\right), \tag{2}$$



(a)



(b)

Figure 1: Daily cumulative number of COVID-19 cases in (a) Spain during period: 25 February - 20 June 2020; and (b) Italy during period: 20 February - 20 June 2020.

where $0 \leq p \leq 1$. The parameter p is introduced to capture different early stages of the epidemic, which can be constant incidence $(p = 0)$, polynomial or sub-exponential growth $(0 < p < 1)$ and exponential growth $(p = 1)$. The GRM has been succesfully applied to model the epidemic growth of, e.g., Zika dan Ebola [36], [37], foot-and-mouth disease [38], and COVID-19 [39]. In those cases, it has been shown that the GRM is a very suitable model for predicting short-term epidemics, including after the epidemic peak.

3.2. Parameter Estimation with Quantified Uncertainty

The next step is to estimate the model parameters by fitting the data to the GRM. Based on the daily cumulative number of cases, we first find a set of parameters $\{\hat{r}, \hat{p}, \hat{K}, \hat{a}\}$ which minimizes the sum of squared-residual between solutions of the GRM and the corresponding observation data, i.e., the model parameters are estimated through a nonlinear curve-fitting in a least-square sense. In Matlab, it can be performed by applying the *lsqcurvefit* function.

The GRM is solved by the fourth-order Runge-Kutta method using the first reported data point as the initial value $(C(0))$. Then we quantify the parameter uncertainty based on a parametric bootstrapping method. A detailed procedure of such quantification of the parameter estimation can be found in [40]. Here, we create M simulated data sets of daily new cases around the previous best-fit model. In this study, the number of simulated data sets is taken to be $M = 200$, where the error is assumed to follow a Poisson distribution centred at the mean at the same points. From each simulated data set of daily new cases, we can create a simulated data set of daily cumulative number of cases. Then, we re-estimate parameters by fitting each of the M data sets to the GRM, based on which we characterize the empirical distributions of parameters and construct a 95% confidence interval.

3.3. Model Calibration and Forecasts

To see the fitting performance, we perform several simulations to calibrate the GRM fit as well as generate an ensemble of epidemic curves for forecasting directly from the uncertainty parameters. The calibration is done for different data intervals. For each simulation we provide 10-day ahead forecasts using the ensemble realization model. For both calibration and forecasting, the reported data and predicted values are compared. In particular we calculate the root-mean-squared-error (RMSE) about the mean and the 95% confidence limit.

4. Results and Discussion

4.1. COVID-19 in Spain

The daily data of total confirmed cases in Spain, see Fig. 1(a), are fitted to the GRM to estimate the model parameters. To verify whether or not they are identifiable from the actual data, we made several parameter estimates with quantified uncertainty for four different data periods, namely 25 February - 15 April 2020 (data of 51 days), 25 February - 15 May 2020 (data of 81 days), 25 February - 31 May 2020 (data of 97 days) and 25 February - 10 June 2020 (data of 107 days). Figures 2 and 3 respectively show the empirical distributions of r, p and K, a of the GRM obtained by 200 bootstrap realizations for the different data periods. For each figure, the best fit parameter and the 95% confidence interval is shown. It is found that using 51 days of data (period 25 February - 15 April 2020), the estimated parameters are considered to be not reliable as the confidence intervals for all parameters are relatively wide. On the contrary, using 81, 97 and 107 days of data, the empirical distributions of all parameters seem to be stable. Furthermore, parameter estimation using more data reduces their uncertainty. Such a phenomenon can be seen from the narrower confidence intervals when more data are used to estimate the GRM parameters.

Using the set of estimated parameters, some simulations are performed to generate an ensemble of epidemic curves. In Fig. 4 we show 10-day GRM ahead forecasts of new daily cases in Spain generated on 15 April 2020, 15 May 2020, 31 May 2020 and 10 June 2020, respectively. In these pictures we plot the mean model solutions and the 95% confidence limits. The 95% confidence limit is very narrow and cannot be observed within the scale of the figure. The calibration period (left) and the forecasting one (right) are separated by a vertical blue-dashed line. For comparison, we also plot the reported new daily cases in Spain. It is observed that the predictions of the GRM using the four different data intervals are reasonably well during the calibration periods. However, the 10-day ahead forecast using 51 days data (25 February - 15 April 2020) has a quite large deviation from the actual data. Such a discrepancy can also be seen from the estimated final epidemic size, which in this case is $K = 2.1164 \times 10^5$ (see Fig. 3), whereas the actual final epidemic size (assuming that there is only a single wave epidemic) is about 3×10^5 (see Fig. 1(a)). In the scale of Fig. 4, it is seen that the other 10-day ahead forecasts using 81, 97 and 107 days of data are equally well. To see the performance of the GRM during the calibration and forecasting periods, we plot in Fig. 5 the RMSE of the GRM prediction for the increasing amount of data. The forecasting error decreases with the length of data period.

According to the parametric bootstrapping approach using 107 days of data, we get the best model parameters: $r = 1.065, p = 0.9292, K = 293220$ and $a = 0.1454$. The 'deceleration of growth' parameter $p = 0.9292 \approx 1$ means that the early epidemic growth profile follows an almost exponential growth dynamics, see [34] for further detail about growth profile. The final size of COVID-19 epidemic in Spain is estimated to be $K = 293220$. The confirmed cumulative number of COVID-19 cases in Spain on 20 June 2020 is 293018. Hence, the COVID-19 epidemic in Spain is predicted to be at the ending stage, see also Fig. 4(d). We also identify that the epidemic peak has already occurred on 2 April 2020. The same peak time of COVID-19 in Spain is also predicted by the GRM fit using data of 81 and 97 days.

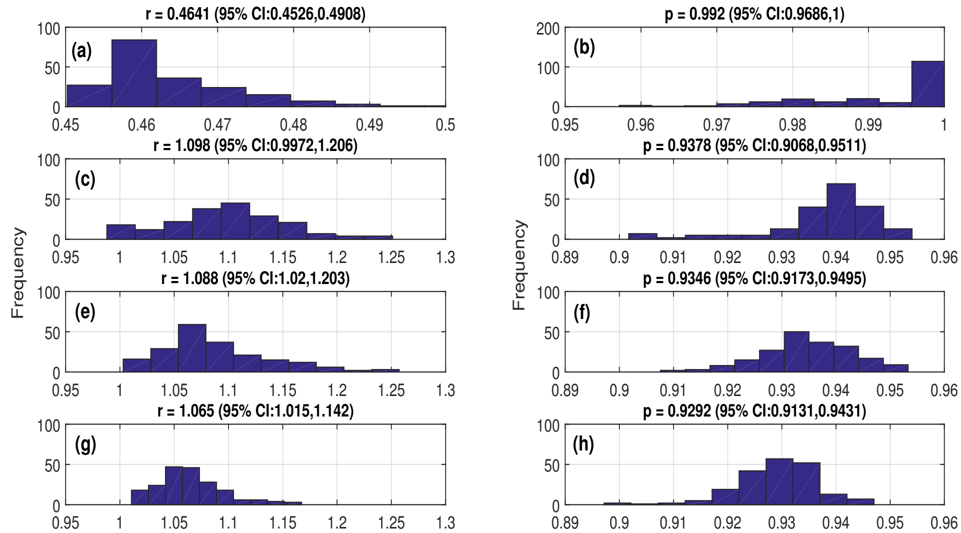


Figure 2: Empirical distributions and 95% confidence intervals for r (left panels) and p (right panels) of the GRM obtained by the parametric bootstrapping with a Poisson error structure for COVID-19 in Spain using data period: (a–b) 25 Feb - 15 Apr 2020 (data 51 days), (c–d) 25 Feb - 15 May 2020 (data 81 days), (e–f) 20 Feb - 31 May 2020 (data 97 days), and (g–h) 20 Feb - 10 Jun 2020 (data 107 days).

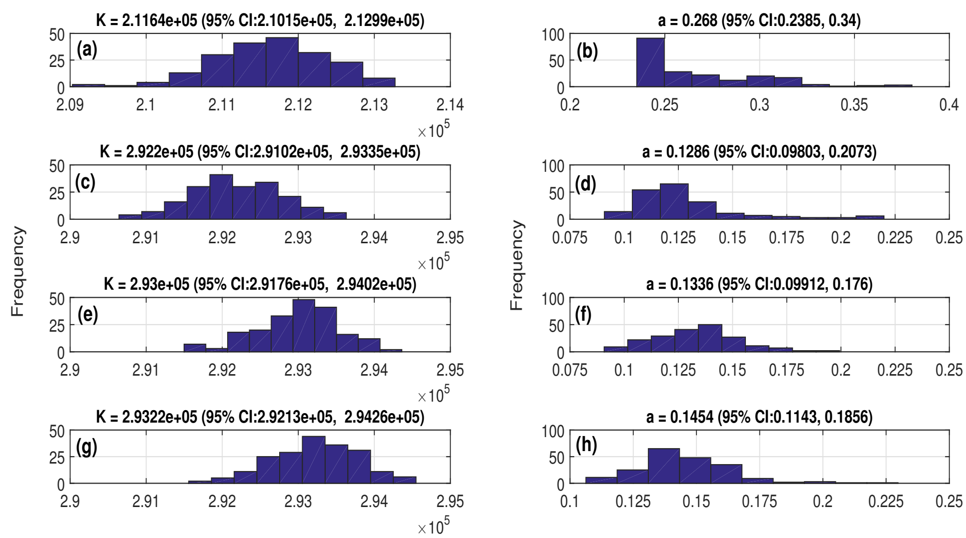


Figure 3: The same as Fig. 2, but for parameters K (left panels) and a (right panels) of the GRM.

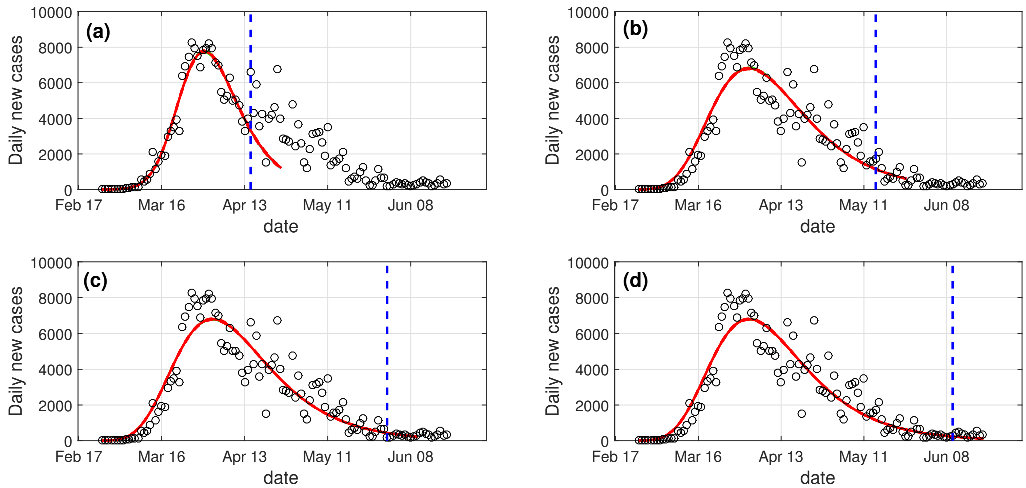


Figure 4: 10-day forecasts based on the GRM for daily new cases of the COVID-19 in Spain using data period: (a) 25 Feb - 15 Apr 2020 (51 days), (b) 25 Feb - 15 May 2020 (81 days), (c) 25 Feb - 31 May 2020 (97 days), and (d) 25 Feb - 10 Jun 2020 (107 days).

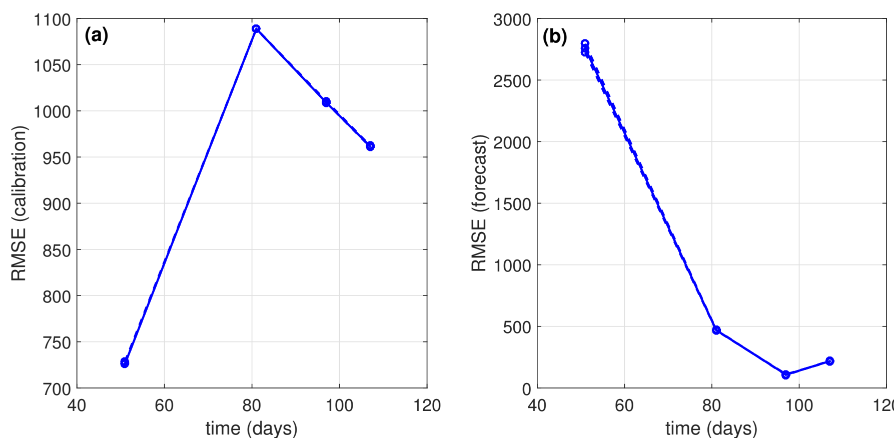


Figure 5: The RMSE for daily new cases in Spain (a) during the calibration and (b) for 10-day forecasts ahead based on the GRM fit when the model is fitted to an increasing amount of epidemic data.

4.2. COVID-19 in Italy

We now turn to Italy, with total confirmed cases shown in Fig. 1(b). As before, we also perform parameter estimation through the GRM (2) with quantified uncertainty for four different data periods, namely the period of 20 Feb–15 Apr 2020 (data of length 56 days), 20 Feb–15 May 2020 (86 days), 25 Feb–31 May 2020 (102 days), and 25 Feb–10 Jun 2020 (112 days). The empirical distributions of the four parameters r, p, K, and a are obtained by 200 bootstrap realizations. The best-fit value for each parameter at the 95% confidence interval is shown in Fig. 6 and 7. It is observed that all the confidence intervals of all parameters obtained by the four different data periods are reasonably narrow. However, the estimated parameter values using data period of 2 Feb - 15 April 2020 are very different from those obtained using the other three data periods. Furthermore, when we apply those sets of parameter values to generate ensembles of epidemic curves using the GRM, it

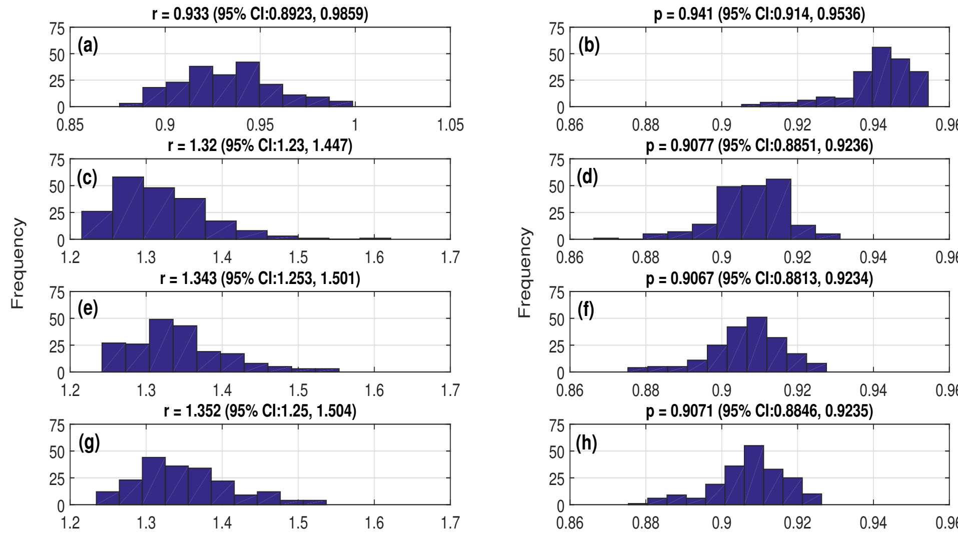


Figure 6: Empirical distributions and 95% confidence intervals for r (left panels) and p (right panels) of the GRM obtained by the parametric bootstrapping with a Poisson error structure for COVID-19 in Italy using data period: (a–b) 20 Feb - 15 Apr 2020 (data of 56 days), (c–d) 20 Feb - 15 May 2020 (86 days), (e–f) 25 Feb - 31 May 2020 (102 days), and (g–h) 25 Feb - 10 Jun 2020 (112 days).

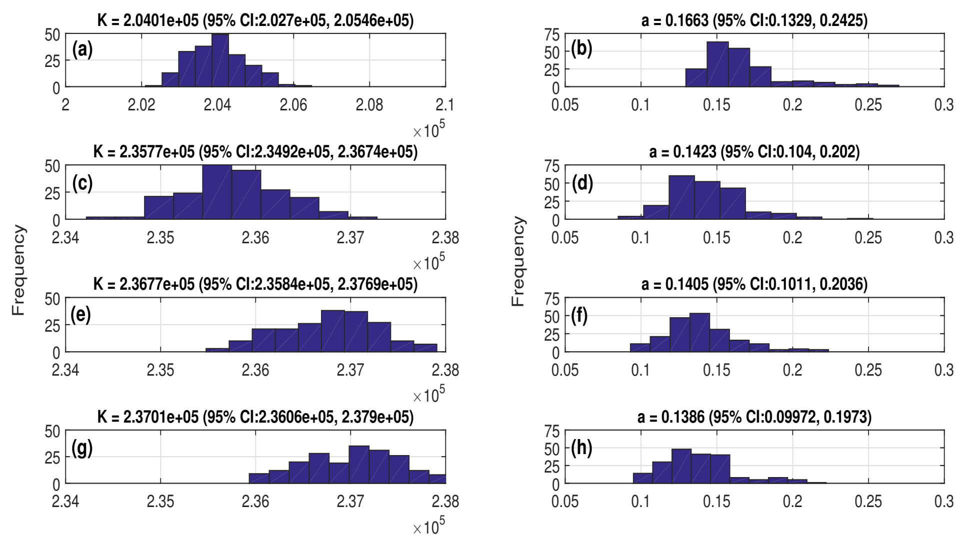


Figure 7: The same as Fig. 6, but for parameters K (left panels) and a (right panels).

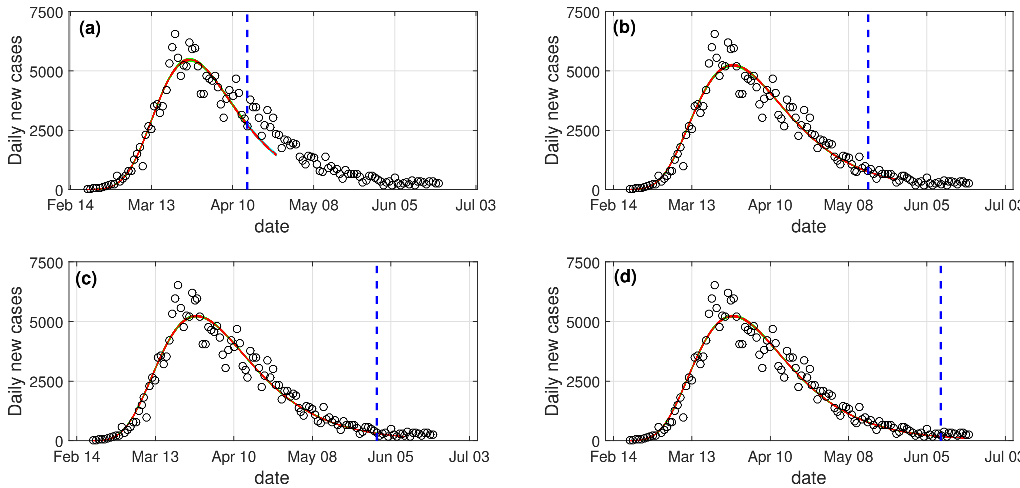


Figure 8: 10-day forecasts based on the GRM for daily new cases of the COVID-19 in Italy using data period: (a) 20 Feb - 15 Apr 2020 (data length of 56 days), (b) 20 Feb - 15 May 2020 (86 days), (c) 20 Feb - 31 May 2020 (102 days), and (d) 20 Feb - 10 Jun 2020 (112 days).

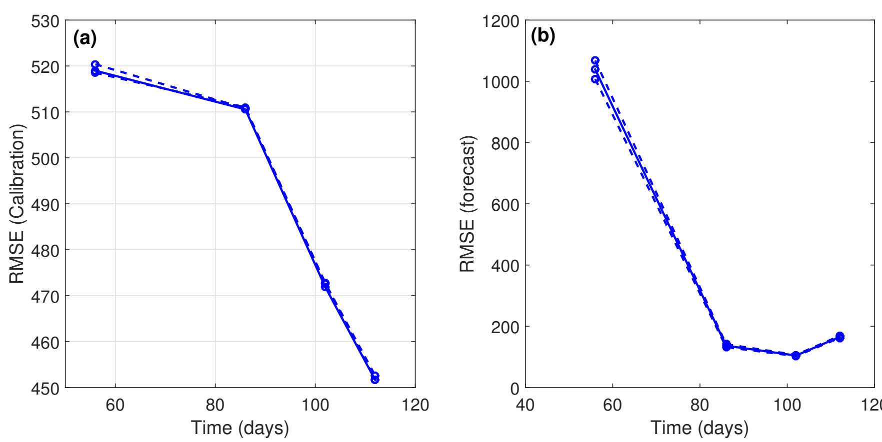


Figure 9: The RMSE for daily new cases in Italy (a) during the calibration and (b) for 10-day forecasts ahead based on the GRM fit with the increasing amount of epidemic data.

is found that the predictions during the calibration period for all sets of parameter values agree well with the reported daily data, see Fig. 8. However, as seen in Fig. 8, the resulting 10-day ahead forecasting of new daily cases in Italy using the period of 20 Feb–15 Apr 2020 (data length of 56 days) deviates significantly from the reported data. On the other hand, the 10-day ahead forecasts using 86, 102 and 112 days of data are equally well. To assess the forecasting performance, we plot in Fig. 9 the RMSE during the calibration and forecasting intervals using the GRM when the model is fitted to the increasing amount of epidemic data (56, 86, 102 and 112 days).

In a similar trend as in the case of COVID-19 in Spain, we see that the error of 10-day ahead forecasting decreases with the increasing length of data period. It can be observed that the 10-day ahead forecasting using 112 days data (25 Feb-10 June 2020) has the best agreement with the data and gives the best parameter values, namely $r = 1.352, p = 0.9071, K = 237010$, and $a = 0.1386$. This result implies that the epidemic growth profile at the early stage also follows an almost exponentially growth (since $p = 0.9071 \approx 1$), and the final size of COVID-19 epidemic in Italy is estimated to be $K = 237010$. From the simulations, we also found that the GRM fit using data of 86, 102 and 112 days shows that the peak of epidemic of COVID-19 in Italy has already occurred on 28 March 2020.

We notice that the GRM model predicts the COVID-19 outbreak in both Spain and Italy will almost subside by mid-June 2020. But it is worth remembering that by mid-June 2020, the infected subpopulation do not actually go extinct. If the intervention measures by both government and society are weakened, the spread of COVID-19 can increase rapidly and the epidemic can return to a new phase or a second wave of the epidemic could occur.

5. Conclusion and Discussion

We have considered a generalized Richards to fit and forecast the cumulative number of infective cases in the current COVID-19 outbreak, particularly in Spain and Italy. The parameter fits have been combined with an uncertainty analysis using a parametric bootstrapping approach. We have showed that the early stages of COVID-19 epidemic, both in Spain and in Italy, followed an almost exponentially growth and the GRM predicts daily new cases of COVID-19 reasonably well during the calibration periods, provided that the amount of data used for parameter estimations is sufficient.

We remark that our fitting is based on a simple model, namely the GRM. The model describes the change of cumulative cases with time. The new daily cases on a day can be determined by calculating the difference between the cumulative number of cases on that day and that of the previous day. Both predicted cumulative number of cases and daily new cases provide insight into the trend of the COVID-19 epidemic under current intervention measures and what happens if the intervention measures are continued. In other words, the estimated parameters for the GRM illustrate the epidemic progression as described by the reported data. Hence, the parameter estimates are aimed to assess the outcome of intervention measures carried out during the outbreak. These parameters cannot quantify the effect of changing intervention measures. To understand the mechanism of COVID-19 transmission as well as to quantify the effect of intervention measures, one must consider a more complex model, such as a compartmental model.

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