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MAX-PLUS ALGEBRA MODEL ON INAPORTNET SYSTEM SHIPS SERVICE SCHEME

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Abstract. *Queues are often found in public service providers, including port services. Therefore, the InaPortNet scheme was developed to facilitate and minimize queues in and out of ships. Also, service performance was optimized through behavior analysis and queuing system stability. This study focused on designing the max-plus algebra model and time estimation on the incoming ship service scheme. The results showed that the InaPortNet system is useful for modeling. Furthermore, the max-plus algebra matrix is used to obtain an estimated service time from registration to the ship docking process at the port. However, further study needs to be carried out by scheduling the max-plus algebraic matrix in order to analyze the behavior and stability of the queuing system*

Keywords: *max-plus algebra, inaportnet system, queuing system.*

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1. INTRODUCTION

Queues often found at public service providers are caused by more visitors in need of services than the officers. This has led to long waiting times for the users, which is detrimental because a lot of time is wasted. Similarly, the service providers also suffer losses due to the reduction in work efficiency, small profit, and bad image created for the customer [1]. In anticipating the occurrence of long queues, service providers' performance needs to be optimized to ensure smooth and effective running.

Queuing situations are often encountered in the service of incoming and outgoing ships at the port which is one of the supporting facilities for sea transportation routes. For example, Indonesia is an archipelagic country, dominated by territorial waters, thereby having potential for queues hence, logistics distribution and mobilization between regions need to be carried out. This logistics distribution has a very strategic role in economic growth because it is one of the business sectors that contributes to national development. To further enhance smooth running services without long waiting times, port management needs to be carried out professionally, transparently, effectively, and efficiently [2].

Currently, the government has developed an information technology infrastructure in the form of a single web/internet-based electronic service system called InaPortNet, to support port activities quickly and transparently. Furthermore, the system is an open and neutral electronic portal developed to facilitate fast, safe, neutral, and easy exchange of port service through relevant information about government agencies, business entities, and logistics industry players to increase the competitiveness of the Indonesian logistics community [3].

In the InaPortNet system scheme, some stages need to be passed, starting from the input process (ship arrival news) to the output (boat docking). Also, the occurrence of queues is categorized in a Discrete Event System (DES) and is modelled by using max-plus algebra. Class of DES mainly contains a manmade system that is able to control and help overcome these problems [4]. This model is an algebraic structure in which the set of all real numbers is equipped with max and plus operations [5],[6], and [7].

Furthermore, this algebra is used to model and algebraically analyse various problems in rail network systems, scheduling in work networks, simple production systems, and queuing [8], [9] and [7]. In the max-plus algebra, the operation max $(\oplus)$ and plus $(\otimes)$ in conventional algebra refer to addition and multiplication [10]. Let, R represent the real number, then $R_{\max} = R \cup \{-\infty\}$. Suppose $a,b \in R_{\max}$, then we define two operators max $(\oplus)$ and plus $(\otimes)$ as $a \oplus b = \max(a,b)$ and $a \otimes b = a + b$ [7][11][12][9].

The two operators are extended to $m \times n$ matrices of elements of $R_{\max}$. The sum of matrices $A, B \in R_{\max}^{m\times n}$ is defined as $(A \oplus B)_{ij} = (A)_{ij} \oplus (B)_{ij}$ for all i, j. The product of $A \in R_{\max}^{m\times l}$ and $B \in R_{\max}^{l\times n}$ is defined as $(A \otimes B)_{ij} = \bigoplus_{k=1}^{l} (A)_{ik} \otimes (B)_{kj}$ [13] [7].

The previous application of max-plus algebra includes: [13] do research on applications of max-plus algebra to design of flow shop scheduling problems. Max-plus algebra using for supply chain scheduling [14]. [15] do research on the application of petri net and max-plus algrbra for bus sscheduling. Max-plus algebra can be alternative solution for scheduling model design of the crystal sugar production system [16]. [17] use max-plus algebraic for modelling of three crossroad traffic queue systems with one underpass in Yogyakarta. Max-plus algebra using to estimate the optimal time required to manufacture the Kupang ikat woven by [18]. Max-plus algebra in Scheduling Model Development Project [19]. [20] using max-plus algebra for modelling of queuing system for making SIUP of Hazardous Substances.

Meanwhile this study model the InaPortNet system inbound service scheme by using max-plus algebra model. In this research, the author use a petri net to represents the flow of the InaPortNet system. Models generated can be used for analyse the behaviour and stability of queuing system.

2. RESEARCH METHODS

In this research, carried out in several stages as follow:

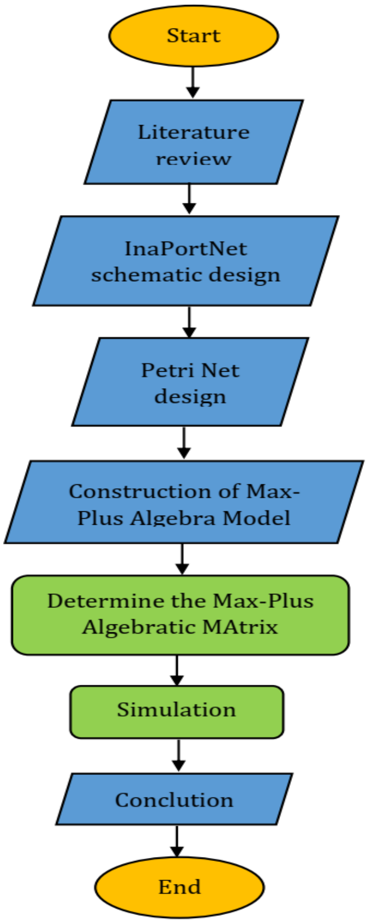

Figure 1. Flowchart of the research

The method used includes the following stages: (1) studying literature related to the InaPortNet system and max-plus algebra, (2) designing a petri net based on the InaPortNet schema flowchart, (3) creating a model max-plus algebra based on the petri net model, (4) determining the max-plus algebraic matrix, and (5) calculating the approximate service time. The software used includes PIPEv4.3.0, Scilab-5.5.2, and Max-Plus Algebra Toolbox ver. 1.0.1 [21].

3. RESULTS AND DISCUSSION

The InaPortNet System Ship Incoming Service Scheme is a business process issued by the Indonesian Ministry of Transportation. According to the flow chart shown in Figure 1, the port user company is assumed to already know the information on the InaPortNet system procedure.

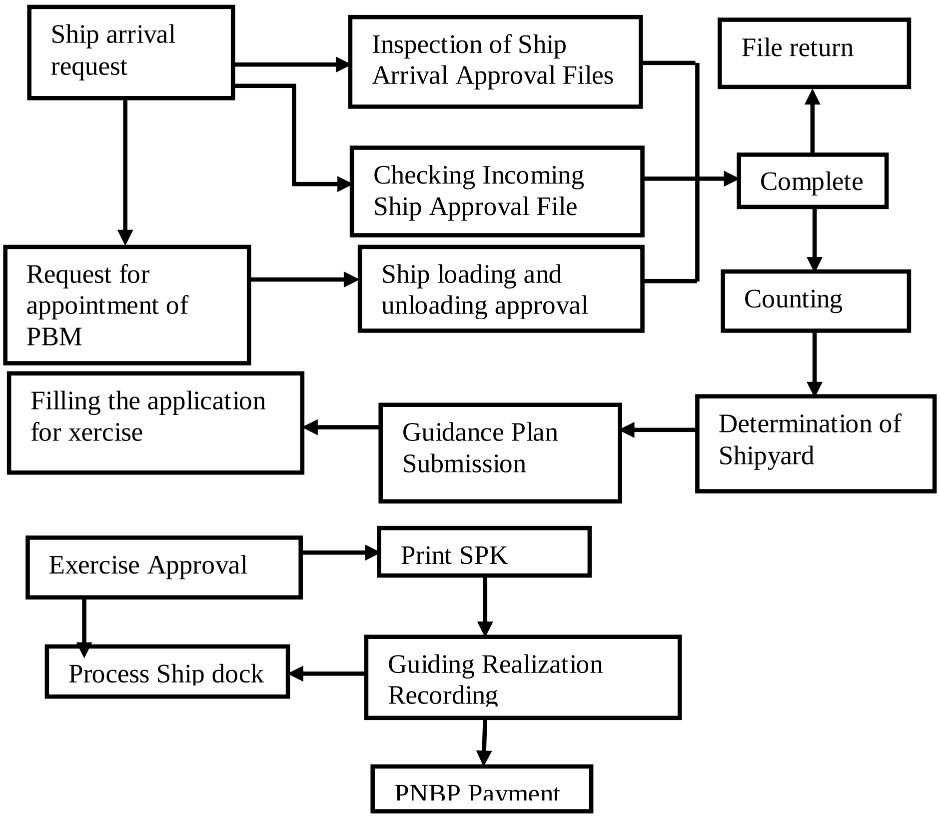

Figure 2. InaPortNet Schematic Port Service Diagram

Prior to the max-plus algebra model, a petri net was designed according to Figure 2, having two variables namely time and length of time indicator [7]. Figure 3 therefore shows the queuing system form of the InaPortNet ship-in service scheme.

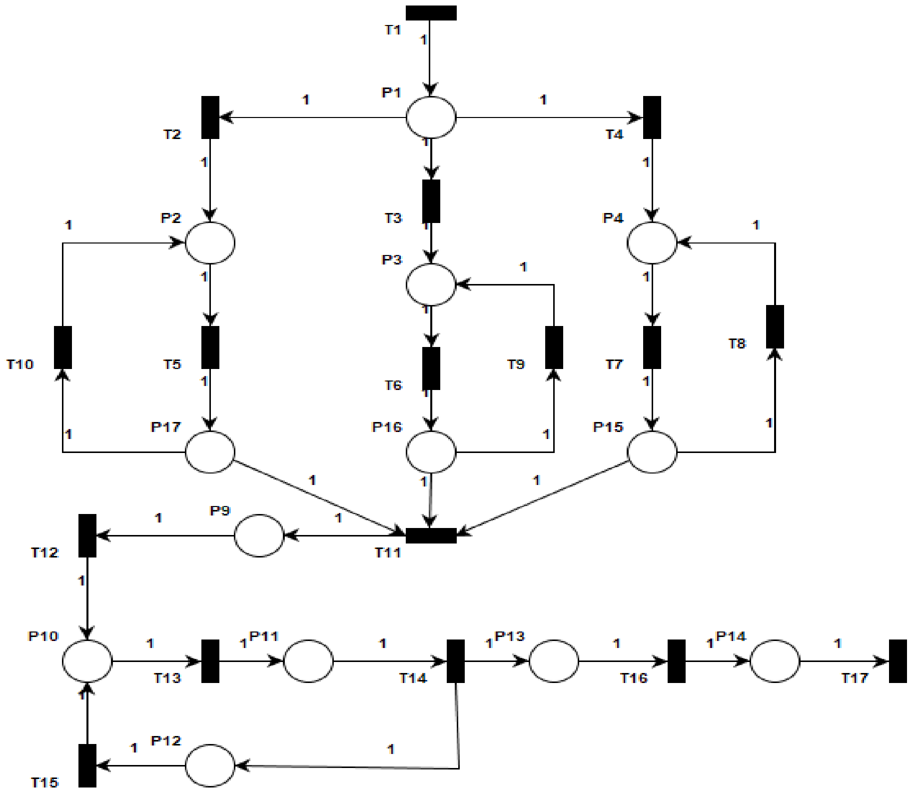

Figure 3. Petri Net Design

The description of the variables showing the time and length of time in Figure 3 is shown in Table 1 and Table 2.

Table 1. Time variables

Variable	Description
$T_1(k)$	Time of when the applicant/ship agent submits the application file for the arrival of the ship when the k-th
$T_2(k)$	Time of checking the completeness of the approval file for the arrival of the ship at the time of the k-th
$T_3(k)$	Time of checking the completeness of the incoming ship approval file at the time of the k-th
$T_4(k)$	Time of checking the completeness of the ship's loading and unloading approval file at the time of the k-th
$T_5(k)$	Time of ship arrival approval file completed on the k-th
$T_6(k)$	Time of the incoming ship approval file is complete on the k-th
$T_7(k)$	Time of the ship loading and unloading approval file is complete at the time of the k-th
$T_8(k)$	Time of the loading and unloading application file is not complete on the k-th
$T_9(k)$	Time of the incoming ship approval file is incomplete on the k-th
$T_{10}(k)$	Time of incomplete ship arrival approval file at the k-th
$T_{11}(k)$	Time of meeting counts at the k-th
$T_{12}(k)$	Time of determination of the ship's berth at the time of the k-th
$T_{13}(k)$	Time of application submission for exercise at the k-th
$T_{14}(k)$	Time of application for exercise is not approved on the k-th
$T_{15}(k)$	Time of Recording of scouting realization on the k-th
$T_{16}(k)$	Time of PNBP payment on the k-th
$T_{17}(k)$	Time of the ship goes to the pier on the k-th

Table 2. Duration of variables

Variable	Description
V_{T_1}, k	Duration of when the applicant/ship agent submits the application file for the arrival of the ship when the k-th
V_{T_1}, k	Duration of checking the completeness of the approval file for the arrival of the ship at the time of the k-th
V_{T_1}, k	Duration of checking the completeness of the incoming ship approval file at the time of the k-th
V_{T_1}, k	Duration of checking the completeness of the ship's loading and Duration of unloading approval file at the time of the k-th
V_{T_1}, k	Duration of ship arrival approval file completed on the k-th
V_{T_1}, k	Duration of the incoming ship approval file is complete on the k-th
V_{T_1}, k	Duration of the ship loading and unloading approval file is complete at the time of the k-th
V_{T_1}, k	Duration of the loading and unloading application file is not complete on the k-th
V_{T_1}, k	Duration of the incoming ship approval file is incomplete on the k-th
V_{T_1}, k	Duration of incomplete ship arrival approval file at the k-th
V_{T_1}, k	Duration of meeting counts at the k-th
V_{T_1}, k	Duration of determination of the ship's berth at the time of the k-th
V_{T_1}, k	Duration of application submission for exercise at the k-th
V_{T_1}, k	Duration of application for exercise is not approved on the k-th
V_{T_1}, k	Duration of recording of scouting realization on the k-th
V_{T_1}, k	Duration of PNBP payment on the k-th
$V_{T_{17}}, k$	Duration of the ship goes to the pier on the k-th

According to Figure 3, the max-plus algebra model is obtained with the time variable as follows:

$$T_1(k) = V_{T_1,k} \otimes T_1(k-1) \quad (1)$$

$$\begin{aligned} T_5(k) &= V_{T_5,k} \otimes \left(T_2(k) \oplus T_{10}(k-1)\right) \\ &= \left(V_{T_5,k} \otimes V_{T_2,k} \otimes V_{T_1,k}\right) \otimes T_1(k-1) \oplus V_{T_5,k} \otimes T_{10}(k-1) \end{aligned} \quad (2)$$

$$T_6(k) = V_{T_6,k} \otimes \left(T_3(k) \oplus T_9(k-1)\right) \\ = \left(V_{T_6,k} \otimes V_{T_3,k} \otimes V_{T_1,k}\right) \otimes T_1(k-1) \oplus V_{T_6,k} \otimes T_9(k-1) \tag{3}$$

$$T_7(k) = V_{T_7,k} \otimes \left(T_4(k) \oplus T_8(k-1)\right) \\ = \left(V_{T_7,k} \otimes V_{T_4,k} \otimes V_{T_1,k}\right) \otimes T_1(k-1) \oplus V_{T_7,k} \otimes T_8(k-1) \tag{4}$$

$$T_{11}(k) = V_{T_{11},k} \otimes \left(T_5(k) \oplus T_6(k) \oplus T_7(k)\right) \\ = \left[\begin{pmatrix} \left(V_{T_{11},k} \otimes V_{T_5,k} \otimes V_{T_2,k} \otimes V_{T_1,k}\right) \oplus \left(V_{T_{11},k} \otimes V_{T_6,k} \otimes V_{T_3,k} \otimes V_{T_1,k}\right) \oplus \\ \left(V_{T_{11},k} \otimes V_{T_7,k} \otimes V_{T_4,k} \otimes V_{T_1,k}\right) \end{pmatrix} \otimes T_1(k-1)\right] \oplus \\ \left[\left(V_{T_{11},k} \otimes V_{T_5,k}\right) \otimes T_{10}(k-1)\right] \oplus \left[\left(V_{T_{11},k} \otimes V_{T_6,k}\right) \otimes T_9(k-1)\right] \oplus \left[\left(V_{T_{11},k} \otimes V_{T_7,k}\right) \otimes T_8(k-1)\right] \tag{5}$$

$$T_{12}(k) = V_{T_{12},k} \otimes T_{11}(k) \\ = \left[\begin{pmatrix} \left(V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_5,k} \otimes V_{T_2,k} \otimes V_{T_1,k}\right) \oplus \\ \left(V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_6,k} \otimes V_{T_3,k} \otimes V_{T_1,k}\right) \oplus \\ \left(V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_7,k} \otimes V_{T_4,k} \otimes V_{T_1,k}\right) \end{pmatrix} \otimes T_1(k-1)\right] \oplus \\ \left[\left(V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_5,k}\right) \otimes T_{10}(k-1)\right] \oplus \\ \left[\left(V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_6,k}\right) \otimes T_9(k-1)\right] \oplus \\ \left[\left(V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_7,k}\right) \otimes T_8(k-1)\right] \tag{6}$$

$$T_{13}(k) = V_{T_{13},k} \otimes \left(T_{12}(k) \oplus T_{15}(k-1)\right) \\ = \left[\begin{pmatrix} \left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_5,k} \otimes V_{T_2,k} \otimes V_{T_1,k}\right) \oplus \\ \left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_6,k} \otimes V_{T_3,k} \otimes V_{T_1,k}\right) \oplus \\ \left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_7,k} \otimes V_{T_4,k} \otimes V_{T_1,k}\right) \end{pmatrix} \otimes T_1(k-1)\right] \oplus \\ \left[\left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_5,k}\right) \otimes T_{10}(k-1)\right] \oplus \\ \left[\left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_6,k}\right) \otimes T_9(k-1)\right] \oplus \\ \left[\left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_7,k}\right) \otimes T_8(k-1)\right] \oplus \left[V_{T_{13},k} \otimes T_{15}(k-1)\right] \tag{7}$$

$$T_{14},(k) = V_{T_{14},k} \otimes T_{13}(k) \\ = \left[\begin{pmatrix} \left(V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_5,k} \otimes V_{T_2,k} \otimes V_{T_1,k}\right) \oplus \\ \left(V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_6,k} \otimes V_{T_3,k} \otimes V_{T_1,k}\right) \oplus \\ \left(V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_7,k} \otimes V_{T_4,k} \otimes V_{T_1,k}\right) \end{pmatrix} \otimes T_1(k-1)\right] \oplus \\ \left[\left(V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_5,k}\right) \otimes T_{10}(k-1)\right] \oplus \\ \left[\left(V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_6,k}\right) \otimes T_9(k-1)\right] \oplus \\ \left[V_{T_{14},k} \otimes \left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_7,k}\right) \otimes T_8(k-1)\right] \oplus \\ \left[V_{T_{14},k} \otimes V_{T_{13},k} \otimes T_{15}(k-1)\right] \tag{8}$$

$$
\begin{aligned}
T_{15}(k) &= V_{T_{15},k} \otimes T_{14}(k) \\
&= \left[\left(\begin{array}{l}
\left(V_{T_{15},k} \otimes \left(V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{5},k} \otimes V_{T_{2},k} \otimes V_{T_{1},k}\right)\oplus\right. \\
\left(V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{6},k} \otimes V_{T_{3},k} \otimes V_{T_{1},k}\right)\oplus \\
\left(V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{7},k} \otimes V_{T_{4},k} \otimes V_{T_{1},k}\right)
\end{array}\right) \otimes T_1(k-1)\right] \oplus \\
&\left[\left(\left(V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{5},k}\right) \otimes T_{10}(k-1)\right)\right] \oplus \\
&\left[\left(\left(V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{6},k}\right) \otimes T_{9}(k-1)\right)\right] \oplus \\
&\left[\left(V_{T_{15},k} \otimes V_{T_{14},k} \otimes \left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{7},k}\right) \otimes T_{8}(k-1)\right)\right] \oplus \\
&\left[\left(V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes T_{15}(k-1)\right)\right]
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
T_{16}(k) &= V_{T_{16},k} \otimes T_{15}(k) \\
&= \left[\left(\begin{array}{l}
\left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{5},k} \otimes V_{T_{2},k} \otimes V_{T_{1},k}\right)\oplus \\
\left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{6},k} \otimes V_{T_{3},k} \otimes V_{T_{1},k}\right)\oplus \\
\left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{7},k} \otimes V_{T_{4},k} \otimes V_{T_{1},k}\right)
\end{array}\right) \otimes T_1(k-1)\right] \oplus \\
&\left[\left(\left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{5},k}\right) \otimes T_{10}(k-1)\right)\right] \oplus \\
&\left[\left(\left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{6},k}\right) \otimes T_{9}(k-1)\right)\right] \oplus \\
&\left[\left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes \left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{7},k}\right) \otimes T_{8}(k-1)\right)\right] \oplus \\
&\left[\left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes T_{15}(k-1)\right)\right]
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
T_{17}(k) &= V_{T_{17},k} \otimes T_{16}(k) \\
&= \left[\left(\begin{array}{l}
\left(V_{T_{17},k} \otimes V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{5},k} \otimes V_{T_{2},k} \otimes V_{T_{1},k}\right)\oplus \\
\left(V_{T_{17},k} \otimes V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{6},k} \otimes V_{T_{3},k} \otimes V_{T_{1},k}\right)\oplus \\
\left(V_{T_{17},k} \otimes V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{7},k} \otimes V_{T_{4},k} \otimes V_{T_{1},k}\right)
\end{array}\right) \otimes T_1(k-1)\right] \oplus \\
&\left[\left(V_{T_{17},k} \otimes \left(V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{5},k}\right) \otimes T_{10}(k-1)\right)\right] \oplus \\
&\left[\left(\left(V_{T_{17},k} \otimes V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{6},k}\right) \otimes T_{9}(k-1)\right)\right] \oplus \\
&\left[\left(V_{T_{17},k} \otimes V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes \left(V_{T_{13},k} \otimes V_{T_{12},k} \otimes V_{T_{11},k} \otimes V_{T_{7},k}\right) \otimes T_{8}(k-1)\right)\right] \oplus \\
&\left[\left(V_{T_{17},k} \otimes V_{T_{16}k} \otimes V_{T_{15},k} \otimes V_{T_{14},k} \otimes V_{T_{13},k} \otimes T_{15}(k-1)\right)\right]
\end{aligned} \tag{11}
$$

Based on equation (1)-(11), we obtain max-plus algebra model into matrix shown in Equation (12).

$$
\begin{pmatrix} T_1(k) \\ T_5(k) \\ T_6(k) \\ T_7(k) \\ T_{11}(k) \\ T_{12}(k) \\ T_{13}(k) \\ T_{14}(k) \\ T_{15}(k) \\ T_{16}(k) \\ T_{17}(k) \end{pmatrix}
=
\begin{pmatrix}
V_{T_1,k} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
a & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & V_{T_5,k} & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
b & \varepsilon & \varepsilon & \varepsilon & \varepsilon & V_{T_6,k} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
c & \varepsilon & \varepsilon & \varepsilon & V_{T_7,k} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
g & \varepsilon & \varepsilon & \varepsilon & d & e & f & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
h & \varepsilon & \varepsilon & \varepsilon & k & j & i & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\
l & \varepsilon & \varepsilon & \varepsilon & o & b & m & \varepsilon & \varepsilon & \varepsilon & V_{T_{13},k} \\
p & \varepsilon & \varepsilon & \varepsilon & s & r & q & \varepsilon & \varepsilon & \varepsilon & t \\
u & \varepsilon & \varepsilon & \varepsilon & x & w & v & \varepsilon & \varepsilon & \varepsilon & y \\
z & \varepsilon & \varepsilon & \varepsilon & ac & ab & aa & \varepsilon & \varepsilon & \varepsilon & ad \\
ae & \varepsilon & \varepsilon & \varepsilon & af & ag & ah & \varepsilon & \varepsilon & \varepsilon & ai
\end{pmatrix}
\otimes
\begin{pmatrix} T_1(k-1) \\ T_5(k-1) \\ T_6(k-1) \\ T_7(k-1) \\ T_8(k-1) \\ T_9(k-1) \\ T_{10}(k-1) \\ T_{12}(k-1) \\ T_{13}(k-1) \\ T_{14}(k-1) \\ T_{15}(k-1) \end{pmatrix} \tag{12}
$$

Where:

$a = V_{T_5,k} \otimes V_{T_2,k} \otimes V_{T_1,k}$ $b = V_{T_6,k} \otimes V_{T_3,k} \otimes V_{T_1,k}$ $c = V_{T_7,k} \otimes V_{T_4,k} \otimes V_{T_1,k}$

$d = V_{T_{11},k} \otimes V_{T_7,k}$ $e = V_{T_{11},k} \otimes V_{T_6,k}$ $f = V_{T_{11},k} \otimes V_{T_5,k}$

$g = V_{T_{11},k} \otimes (a \oplus b \oplus c)$ $h = V_{T_{12},k} \otimes g$ $i = V_{T_{12},k} \otimes f$

$j = V_{T_{12},k} \otimes e$ $k = V_{T_{12},k} \otimes d$ $l = V_{T_{13},k} \otimes h$

$m = V_{T_{13},k} \otimes i$ $n = V_{T_{13},k} \otimes j$ $o = V_{T_{13},k} \otimes k$

$p = V_{T_{14},k} \otimes l$ $q = V_{T_{14},k} \otimes m$ $r = V_{T_{14},k} \otimes n$

$s = V_{T_{14},k} \otimes o$ $t = V_{T_{14},k} \otimes V_{T_{13},k}$ $u = V_{T_{15},k} \otimes p$

$v = V_{T_{15},k} \otimes q$ $w = V_{T_{15},k} \otimes r$ $x = V_{T_{15},k} \otimes s$

$y = V_{T_{15},k} \otimes t$ $z = V_{T_{16},k} \otimes u$ $aa = V_{T_{16},k} \otimes v$

$ab = V_{T_{16},k} \otimes w$ $ac = V_{T_{16},k} \otimes x$ $ad = V_{T_{16},k} \otimes y$

$ae = V_{T_{17},k} \otimes z$ $af = V_{T_{17},k} \otimes aa$ $ag = V_{T_{17},k} \otimes ab$

$ah = V_{T_{17},k} \otimes ac$ $ai = V_{T_{17},k} \otimes ad$

Furthermore, the processing time for each stage in the service is given as follows:

$V_{T_1,k} = 10$ $V_{T_2,k} = 10$ $V_{T_3,k} = 10$ $V_{T_4,k} = 10$ $V_{T_5,k} = 15$ $V_{T_6,k} = 15$

$V_{T_7,k} = 15$ $V_{T_8,k} = 5$ $V_{T_9,k} = 5$ $V_{T_{10},k} = 5$ $V_{T_{11},k} = 20$ $V_{T_{12},k} = 10$

$V_{T_{13},k} = 10$ $V_{T_{14},k} = 5$ $V_{T_{15},k} = 10$ $V_{T_{16},k} = 20$ $V_{T_{17},k} = 20$

Obtained:

$$\begin{pmatrix} T_1(k) \\ T_5(k) \\ T_6(k) \\ T_7(k) \\ T_{11}(k) \\ T_{12}(k) \\ T_{13}(k) \\ T_{14}(k) \\ T_{15}(k) \\ T_{16}(k) \\ T_{17}(k) \end{pmatrix} = \begin{pmatrix} 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 35 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 35 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 35 & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 55 & \varepsilon & \varepsilon & \varepsilon & 35 & 35 & 35 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 65 & \varepsilon & \varepsilon & \varepsilon & 45 & 45 & 45 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 75 & \varepsilon & \varepsilon & \varepsilon & 55 & 55 & 55 & \varepsilon & \varepsilon & \varepsilon & 10 \\ 80 & \varepsilon & \varepsilon & \varepsilon & 60 & 60 & 60 & \varepsilon & \varepsilon & \varepsilon & 15 \\ 90 & \varepsilon & \varepsilon & \varepsilon & 70 & 70 & 70 & \varepsilon & \varepsilon & \varepsilon & 25 \\ 110 & \varepsilon & \varepsilon & \varepsilon & 90 & 90 & 90 & \varepsilon & \varepsilon & \varepsilon & 45 \\ 130 & \varepsilon & \varepsilon & \varepsilon & 110 & 110 & 110 & \varepsilon & \varepsilon & \varepsilon & 65 \end{pmatrix} \otimes \begin{pmatrix} T_1(k-1) \\ T_5(k-1) \\ T_6(k-1) \\ T_7(k-1) \\ T_8(k-1) \\ T_9(k-1) \\ T_{10}(k-1) \\ T_{12}(k-1) \\ T_{13}(k-1) \\ T_{14}(k-1) \\ T_{15}(k-1) \end{pmatrix} \quad (13)$$

If initial state from Equation (13) is ....

$$\begin{pmatrix} T_1(k) \\ T_5(k) \\ T_6(k) \\ T_7(k) \\ T_{11}(k) \\ T_{12}(k) \\ T_{13}(k) \\ T_{14}(k) \\ T_{15}(k) \\ T_{16}(k) \\ T_{17}(k) \end{pmatrix} = \begin{pmatrix} 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 35 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 35 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 35 & \varepsilon & \varepsilon & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 55 & \varepsilon & \varepsilon & \varepsilon & 35 & 35 & 35 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 65 & \varepsilon & \varepsilon & \varepsilon & 45 & 45 & 45 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 75 & \varepsilon & \varepsilon & \varepsilon & 55 & 55 & 55 & \varepsilon & \varepsilon & \varepsilon & 10 \\ 80 & \varepsilon & \varepsilon & \varepsilon & 60 & 60 & 60 & \varepsilon & \varepsilon & \varepsilon & 15 \\ 90 & \varepsilon & \varepsilon & \varepsilon & 70 & 70 & 70 & \varepsilon & \varepsilon & \varepsilon & 25 \\ 110 & \varepsilon & \varepsilon & \varepsilon & 90 & 90 & 90 & \varepsilon & \varepsilon & \varepsilon & 45 \\ 130 & \varepsilon & \varepsilon & \varepsilon & 110 & 110 & 110 & \varepsilon & \varepsilon & \varepsilon & 65 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ 35 \\ 35 \\ 35 \\ 555 \\ 65 \\ 75 \\ 80 \\ 90 \\ 110 \\ 130 \end{pmatrix} \quad (14)$$

According to the results, obtained that $T_1(1) = 10$ indicated that the time required for the ship agent to apply for the first arrival is 10 minutes. Also, $T_{17}(1) = 130$ showed that the ship agent had succeeded in conveying arrival information until the ship docked at the port 130 minutes after the arrival time.

4. CONCLUSIONS

InaPortNet system inbound service schema is modeled by using max-plus algebra and the result is used to analyze the systems' behavior and stability. Therefore, the estimated time required for incoming ship services, starting from the delivery of arrival information to the ship docking process at the port, is 130 minutes.

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