



Goodwin Model with Clustering Workers' Skills in Indonesian Economic Cycle

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ABSTRACT

The economic model which deals with the economic cycle is Goodwin's Model. It presents the relationship between employment rate and wage shares. In this study, the modification model was made taking into three types of workers, namely high-, medium-, and low-skilled workers. Studies of model are conducted by determining the equilibrium point and its stability analysis. Furthermore, a numerical simulation is given to see which model satisfy the ideal of Goodwin's model cycle prediction by using Indonesian data from 2000 to 2020. At the end, an investigation into the effects of reducing the wage gap between the three types of workers was conducted. The results showed two equilibrium points, namely The Equilibrium Point without Employment Rate and The Wages Share (T_1) and the Existence Equilibrium Point of Employment Rate and Wages Share (T_2). T_1 achieves a stable node locally condition when $S_c\sigma < \delta + \phi + n$ while T_2 reaches a center locally condition when $S_c\sigma > \delta + \phi + n$. The simulation showed Goodwin's model of low-skilled workers produced the ideal of Goodwin model cycle whereas Goodwin's model of high-, medium-skilled workers, and the entire economy (capitalist) didn't produce the ideal of Goodwin model cycle. Eventually the effects of reducing the wage gap makes the economy unstable.

Keywords: Goodwin's Model; Economic Cycles; Clustering the Workers Skills

INTRODUCTION

Mathematical modeling is the process of differencing the mathematical model of a phenomenon based on the assumptions used. The purpose of a mathematical model is to enable a decision-making process about real situations by analyzing the model [1]. The role of mathematical models is widely applied to various cases, including in the fields of health, biology, economics and others [2]–[7]. One of the topics that can be studied in prospective mathematical modeling in economics is the problem of economic cycles.

The economic cycle is a wave of ups and downs of a country's economic activities and occurs over and over again [8]. The current economic situation makes it possible to

achieve wage determination in appropriate with the conditions of a country and the capabilities of the company, so it is necessary to determine the minimum wage that concerns the fulfillment of the Needs of Decent Living (NDL). The government sets wages in accordance with the NDL, by observing the production power and economic development [9]. Wage determination must be adjusted to the types of skills of workers. In general, workers' skills are divided into high- and low-skilled workers. To make adjustments and approaches to real phenomena, the research is needed on workers who have skills between high and low-skilled workers, namely medium-skilled workers. So this study was conducted on the skills of high, medium, and low-skilled workers and investigated its impact in the Indonesian economic cycle.

A mathematical model that predicts the existence of an economic cycle and shows fluctuations in an economic state is Goodwin's Class Struggle Model or better known as the Goodwin Model [10]. This model is widely studied from time to time, especially in its application in economic problems [11]. This model continues to develop and modify over the time. Goodwin's model only considers the employment rates and wage share, so Grasselli and Lima [12] developed their research by analyzing Goodwin's model by including debt-financed investments and this model was called the Keen model. Furthermore, Goodwin's model and Keen's model were analyzed for Minsky's financial instability hypothesis model. Sasaki et al., [13] developed a macro dynamic model taking into account two types of labor (regular and non-regular workers) and investigating how the expansion of the wage gap between regular and non-regular workers affected the economy and analyzed how the economic cycle was affected by the introduction of the minimum wage. Sasaki and Asada [14] extended Goodwin's model by considering two types of workers: low and high-skilled workers at the employment rate and the wages share. Sasaki and Asada [14] investigated how the introduction of minimum wage share affected employment rates and the wage share as well as the effect of reducing the wage gap between low and high-skilled workers using 1989-2018 Japanese economic data. The research showed the introduction of minimum wage share reduced the amplitude of fluctuations and stabilized the economy. The effect of reducing the wage gap between low- and high-skilled workers increases the amplitude of fluctuations of the wage share and employment rates.

In this study, a model modification was made that refers to Sasaki and Asada [14] by adding the type of workers to three types of workers, namely high-skilled, medium-skilled and low-skilled workers. Furthermore, the analysis is carried out by determining the fixed point of the model along with its stability and numerical simulation by applying Indonesian economic data in 2000-2020 [15] in the case of the Indonesian economic cycle to see which model satisfy the ideal of Goodwin model cycle predictions. At the end, an investigation into the effects of reducing the wage gap between high-skilled, medium-skilled, and low-skilled workers was conducted to see if the reducing the wage gap could stabilize the economic cycle.

METHODS

This research was conducted using the study literature method with the following steps:

1. Literature review related to Goodwin model and Indonesia's economic cycle.
2. Modifying Goodwin's model with three types of worker skills.
3. Determining the equilibrium point stability analysis modifying the Goodwin model with three types of worker.

4. Conduct numerical simulations of Goodwin model with three types of worker using Indonesian economic data from 2000 to 2020.
5. Investigate the effects of reducing the wage gap between three types of worker.

RESULTS AND DISCUSSION

Mathematical Model

In this article, mathematical models are formulated based on assumptions below:

1. Goodwin's model is reviewed based on the skill of workers that are high-, medium-, and low-skilled workers and the entire economy (capitalist), where the entire economy (capitalist) is the sum of all workers' skills or representing workers as a whole
2. The measurement of workers' skills is based on the last level of education completed
3. High-skilled workers are workers who have the highest level of education completed is Diploma I/II/III/Academy/University (workers aged 19 years and older)
4. Medium-skilled workers are workers who have the highest level of education that is completed is high school/vocational school (workers aged 17 years and older)
5. Low-skilled workers are workers who have the highest level of education that is completed are junior high school/MTs (workers aged 15 years and older)
6. There are only two factors of production: labor and capital
7. We suppose have an economy in which low-skilled workers, medium-skilled workers, high-skilled workers, and capitalists coexist
8. Firms produce a single good used for both consumption and investment using low-, medium-, and high-skilled workers, (as a labor) and capital stock according to the following Leontief production function
9. Labor productivity of both groups increases at the same constant rate (ϕ)
10. Firms are assumed to adopt cost-minimizing behaviors
11. The labor supply (L) grows at a constant rate (n)
12. The growth rate of the real wage rate of high-skilled workers (w_H) is an increasing function of the employment rate of high-skilled workers x_H
13. The real wage rate of high-skilled workers is more than that of medium-skilled workers by a constant factor and the real wage rate of medium-skilled workers is more than that of low-skilled workers by a constant factor.
14. The low- skilled workers (S_w^L), medium- skilled workers (S_w^M), and high-skilled workers (S_w^H) save their wage incomes at constant rates, respectively, and that capitalists (S_c) save their profit incomes at a constant rate
15. Saving rates $S_w^L < S_w^M < S_w^H < S_c$
16. All the parameters are positive.

Based on the assumptions above, the dynamics of the Goodwin model with three types of worker can be represented in the following set of differential equations divided based on the types of each skill.

Goodwin models of high-skilled workers:

$$\dot{x}_H = \left[S_c \sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) y_H \right] x_H \quad (1)$$

$$\dot{y}_H = -[(\alpha + \phi) - \beta x_H]y_H \quad (2)$$

Goodwin models of medium-skilled workers:

$$\dot{x}_M = \left[S_c \sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\bar{b}\gamma}{\bar{a}} y_M \right] x_M \quad (3)$$

$$\dot{y}_M = - \left[(\alpha + \phi) - \beta \frac{\bar{b}}{\bar{a}} x_M \right] y_M \quad (4)$$

Goodwin models of low-skilled workers:

$$\dot{x}_L = \left[S_c \sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\gamma v}{\bar{a}} y_L \right] x_L \quad (5)$$

$$\dot{y}_L = - \left[(\alpha + \phi) - \beta \frac{1}{\bar{a}} x_L \right] y_L \quad (6)$$

Goodwin model of the entire economy (capitalist):

$$\dot{x} = \left[S_c \sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\bar{b}\gamma v}{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}} y \right] x \quad (7)$$

$$\dot{y} = - \left[(\alpha + \phi) - \beta \frac{\bar{b}}{\bar{b} + \bar{a} + \bar{a}\bar{b}} x \right] y \quad (8)$$

Description:

- x_H : The employment rate of high-skilled workers
- y_H : The wage share of high-skilled workers
- x_M : The employment rate of medium-skilled workers
- y_M : The wage share of medium-skilled workers
- x_L : The employment rate of low-skilled workers
- y_L : The wage share of low-skilled workers
- x : The employment rate of the entire economy (capitalists)
- y : The wage share of the entire economy (capitalists)

The description of all parameters presented in Table 5.

Equilibrium Points and it's Stability Analysis of Goodwin's Model with Three Types of Workers

- The Determination of Equilibrium Points and it's Stability Analysis of Goodwin Model of High-Skilled Workers

The determination of the equilibrium point of Goodwin model of high-skilled workers in equations (1) and (2) is obtained by solving equations $\dot{x}_H = 0$ and $\dot{y}_H = 0$. The results showed two equilibrium points, namely the equilibrium point without employment rate and the wages share (T_1) where $T_1(x_H, y_H) = (0,0)$ and the existence equilibrium point of employment rate and wages share (T_2) where $T_2(x_H^*, y_H^*) = \left(\frac{\alpha + \phi}{\beta}, \frac{(S_c \sigma - \delta - \phi - n) \bar{b} \gamma v}{\sigma (S_c (\bar{b} \gamma v + \bar{a} v + \bar{a} \bar{b}) - S_w^H (\bar{b} \gamma v) - S_w^M (\bar{a} v) - S_w^L (\bar{a} \bar{b}))} \right)$. The equilibrium point without employment rate and the wages share $T_1(x_H, y_H) = (0,0)$ in the economy has no meaning because the employment rates (x_H) and the wages share of high-skilled workers (y_H) are both worth zero. While the existence equilibrium point of employment rate and wages share $T_2(x_H^*, y_H^*) = \left(\frac{\alpha + \phi}{\beta}, \frac{(S_c \sigma - \delta - \phi - n) \bar{b} \gamma v}{\sigma (S_c (\bar{b} \gamma v + \bar{a} v + \bar{a} \bar{b}) - S_w^H (\bar{b} \gamma v) - S_w^M (\bar{a} v) - S_w^L (\bar{a} \bar{b}))} \right)$ has a

good meaning in economically because the employment rates and the wage share of high-skilled workers has positive value.

The determination of stability first lineing the equations (1) and (2). The Jacobian matrix is obtained:

$$J(x_H, y_H) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (9)$$

Where:

$$a = S_c\sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) y_H$$

$$b = -\sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) x_H$$

$$c = y_H \beta$$

$$d = -[(\alpha + \phi) - \beta x_H]$$

The stability of the equilibrium point studied by local condition. It is seen from the eigenvalue that generated by substitute the equilibrium point without employment rate and the wages share $T_1(x_H, y_H)$ and the existence equilibrium point of employment rate and wages share $T_2(x_H^*, y_H^*)$ into the Jacobian matrix $J(x_H, y_H)$ in equation (9), the eigen value of the the equilibrium point $T_1(x_H, y_H)$ follows:

$$\lambda_1 = S_c\sigma - \delta - \phi - n \quad \vee \quad \lambda_2 = -\alpha - \phi$$

To satisfy the stability, all the eigenvalues must be negative. Thus the stability of equilibrium point of $T_1(x_H, y_H)$ is a stable node locally condition if it satisfy $S_c\sigma < \phi + \delta + n$. And the eigen value of the the equilibrium point $T_2(x_H^*, y_H^*)$ follows:

$$\lambda_{12} = \pm i\sqrt{(\alpha + \phi)(S_c\sigma - \delta - \phi - n)}$$

From the eigen value obtained, the stability of equilibrium point of $T_2(x_H^*, y_H^*)$ is a center locally condition when $S_c\sigma > \phi + \delta + n$. The stability of the equilibrium point without employment rate and the wages share $T_1(x_H, y_H)$ and the existence equilibrium point of employment rate and wages share $T_2(x_H^*, y_H^*)$ based on the eigenvalues obtained results such as in Table 1.

Table 1. The Equilibrium Stability of The Goodwin Model of High-Skilled Workers

The Stability	T_1	T_2
$S_c\sigma > \phi + \delta + n$	Saddle	Local center
$S_c\sigma < \phi + \delta + n$	Local stable node	Saddle

- The Determination of Equilibrium Points and it's Stability Analysis of Goodwin Model of Medium-Skilled Workers

The determination of the equilibrium point of Goodwin model of medium-skilled workers in equations (3) and (4) is obtained by solving equations $\dot{x}_M = 0$ and $\dot{y}_M = 0$. The results showed two equilibrium points, namely the equilibrium point without employment rate and the wages share (T_1) where $T_1(x_M, y_M) = (0,0)$ and the existence equilibrium point of employment rate and wages share (T_2) where $T_2(x_M^*, y_M^*) = \left(\frac{(\alpha + \phi)\bar{a}}{\beta b}, \frac{(S_c\sigma - \delta - \phi - n)\bar{a}v}{\sigma(S_c(\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}) - S_w^H(\bar{b}\gamma v) - S_w^M(\bar{a}v) - S_w^L(\bar{a}\bar{b}))} \right)$. The equilibrium point without employment rate and the wages share $T_1(x_M, y_M) = (0,0)$ in the economy has no meaning because the employment rates (x_M) and the wages share of medium-skilled workers (y_M) are both worth zero. While the existence equilibrium point of employment

rate and wages share $T_2(x_M^*, y_M^*) = \left(\frac{(\alpha + \phi)\bar{a}}{\beta b}, \frac{(S_c\sigma - \delta - \phi - n)\bar{a}v}{\sigma(S_c(\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}) - S_w^H(\bar{b}\gamma v) - S_w^M(\bar{a}v) - S_w^L(\bar{a}\bar{b}))} \right)$ has a good meaning in economically because the employment rates and the wage share of medium-skilled workers has positive value.

The determination of stability first lineing the equations (3) and (4). The Jacobian matrix is obtained:

$$J(x_M, y_M) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (10)$$

Where:

$$\begin{aligned} a &= S_c\sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\gamma \bar{b}}{\bar{a}} y_M \\ b &= -\sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\gamma \bar{b}}{\bar{a}} x_M \\ c &= y_M \beta \frac{\bar{b}}{\bar{a}} \\ d &= - \left[(\alpha + \phi) - \beta \frac{\bar{b}}{\bar{a}} x_M \right] \end{aligned}$$

The stability of the equilibrium point studied by local condition. It is seen from the eigenvalue that generated by substitute the equilibrium point without employment rate and the wages share $T_1(x_M, y_M)$ and the existence equilibrium point of employment rate and wages share $T_2(x_M^*, y_M^*)$ into the Jacobian matrix $J(x_M, y_M)$ in equation (10), the eigen value of the the equilibrium point $T_1(x_M, y_M)$ follows:

$$\lambda_1 = S_c\sigma - \delta - \phi - n \quad \vee \quad \lambda_2 = -\alpha - \phi$$

To satisfy the stability, all the eigenvalues must be negative. Thus the stability of equilibrium point of $T_1(x_M, y_M)$ is a stable node locally condition if it satisfy $S_c\sigma < \phi + \delta + n$. And the eigen value of the the equilibrium point $T_2(x_M^*, y_M^*)$ follows:

$$\lambda_{12} = \pm i \sqrt{(\alpha + \phi)(S_c\sigma - \delta - \phi - n)}$$

From the eigen value obtained, the stability of equilibrium point of $T_2(x_M^*, y_M^*)$ center locally condition when $S_c\sigma > \phi + \delta + n$. So that the stability of the equilibrium point without employment rate and the wages share $T_1(x_M, y_M)$ and the existence equilibrium point of employment rate and wages share $T_2(x_M^*, y_M^*)$ based on the eigenvalues obtained results such as in Table 2.

Table 2. The Equilibrium Stability of the Goodwin Model of Medium-Skilled Workers

The Stability	T_1	T_2
$S_c\sigma > \phi + \delta + n$	Saddle	Local Center
$S_c\sigma < \phi + \delta + n$	Local stable node	Saddle

- The Determination of Equilibrium Points and it's Stability Analysis of Goodwin Model of Low-Skilled Workers

The determination of the equilibrium point of Goodwin model of low-skilled workers in equations (5) and (6) is obtained by solving equations $\dot{x}_L = 0$ and $\dot{y}_L = 0$. The results showed two equilibrium points, namely the equilibrium point without employment rate and the wages share (T_1) where $T_1(x_L, y_L) = (0, 0)$ and the existence equilibrium point of employment rate and wages share (T_2) where $T_2(x_L^*, y_L^*) =$

$\left(\frac{(\alpha+\phi)\bar{a}}{\beta}, \frac{(S_c\sigma-\delta-\phi-n)\bar{a}\bar{b}}{\sigma(S_c(\bar{b}\gamma v+\bar{a}v+\bar{a}\bar{b})-S_w^H(\bar{b}\gamma v)-S_w^M(\bar{a}v)-S_w^L(\bar{a}\bar{b}))} \right)$. The equilibrium point without employment rate and the wages share $T_1(x_L, y_L) = (0,0)$ in the economy has no meaning because the employment rates (x_L) and the wages share of low-skilled workers (y_L) are both worth zero. While the existence equilibrium point of employment rate and wages share $T_2(x_L^*, y_L^*) = \left(\frac{(\alpha+\phi)\bar{a}}{\beta}, \frac{(S_c\sigma-\delta-\phi-n)\bar{a}\bar{b}}{\sigma(S_c(\bar{b}\gamma v+\bar{a}v+\bar{a}\bar{b})-S_w^H(\bar{b}\gamma v)-S_w^M(\bar{a}v)-S_w^L(\bar{a}\bar{b}))} \right)$ has a good meaning in economically because the employment rates and the wage share of low-skilled workers has positive value.

The determination of stability first lineing the equations (5) and (6). The Jacobian matrix is obtained:

$$J(x_L, y_L) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (11)$$

Where:

$$\begin{aligned} a &= S_c\sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma\bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\gamma v}{\bar{a}} y_M \\ b &= -\sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma\bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\gamma v}{\bar{a}} x_M \\ c &= y_M \beta \frac{1}{\bar{a}} \\ d &= - \left[(\alpha + \phi) - \beta \frac{1}{\bar{a}} x_M \right] \end{aligned}$$

The stability of the equilibrium point studied by local condition. It is seen from the eigenvalue that generated by substitute the equilibrium point without employment rate and the wages share $T_1(x_L, y_L)$ and the existence equilibrium point of employment rate and wages share $T_2(x_L^*, y_L^*)$ into the Jacobian matrix $J(x_L, y_L)$ in equation (11), the eigen value of the the equilibrium point $T_1(x_L, y_L)$ follows:

$$\lambda_1 = S_c\sigma - \delta - \phi - n \quad \vee \quad \lambda_2 = -\alpha - \phi$$

To satisfy the stability, all the eigenvalues must be negative. Thus the stability of equilibrium point of $T_1(x_L, y_L)$ is a stable node locally condition if it satisfy $S_c\sigma < \phi + \delta + n$. And the eigen value of the the equilibrium point $T_2(x_L^*, y_L^*)$ follows:

$$\lambda_{12} = \pm i\sqrt{(\alpha + \phi)(S_c\sigma - \delta - \phi - n)}$$

From the eigen value obtained, the stability of equilibrium point of $T_2(x_L^*, y_L^*)$ center locally condition when $S_c\sigma > \phi + \delta + n$. So that the stability of the equilibrium point without employment rate and the wages share $T_1(x_L, y_L)$ and the existence equilibrium point of employment rate and wages share $T_2(x_L^*, y_L^*)$ based on the eigenvalues obtained results such as in Table 3.

Table 3. The Equilibrium Stability of the Goodwin Model of Low-Skilled Workers

The Stability	T_1	T_2
$S_c\sigma > \phi + \delta + n$	Saddle	Local center
$S_c\sigma < \phi + \delta + n$	Local stable node	Saddle

- The Determination of Equilibrium Points and it's Stability Analysis of Goodwin Model of The Entire Economy (Capitalists)

The determination of the equilibrium point of Goodwin model of the entire economy (capitalists) in equations (7) and (8) is obtained by solving equations $\dot{x} = 0$ and $\dot{y} = 0$. The results showed two equilibrium points, namely the equilibrium point without employment rate and the wages share (T_1) where $T_1(x, y) = (0,0)$ and the existence

equilibrium point of employment rate and wages share (T_2) where $T_2(x^*, y^*) = \left(\frac{(\alpha + \phi)(\bar{b} + \bar{a} + \bar{a}\bar{b})}{\beta b}, \frac{(S_c \sigma - \delta - \phi - n)(\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b})}{\sigma(S_c(\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}) - S_w^H(\bar{b}\gamma v) - S_w^M(\bar{a}v) - S_w^L(\bar{a}\bar{b}))} \right)$. The equilibrium point without employment rate and the wages share $T_1(x, y) = (0, 0)$ in the economy has no meaning because the employment rates (x) and the wages share of the entire economy (capitalists) (y) are both worth zero. While the existence equilibrium point of employment rate and wages share $T_2(x^*, y^*) = \left(\frac{(\alpha + \phi)(\bar{b} + \bar{a} + \bar{a}\bar{b})}{\beta b}, \frac{(S_c \sigma - \delta - \phi - n)(\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b})}{\sigma(S_c(\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}) - S_w^H(\bar{b}\gamma v) - S_w^M(\bar{a}v) - S_w^L(\bar{a}\bar{b}))} \right)$ has a good meaning in economically because the employment rates and the wage share of the entire economy (capitalists) has positive value.

The determination of stability first lineing the equations (7) and (8). The Jacobian matrix is obtained:

$$J(x, y) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (12)$$

Where:

$$\begin{aligned} a &= S_c \sigma - \delta - \phi - n - \sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\bar{b}\gamma v}{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}} y \\ b &= -\sigma \left(S_c \frac{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}}{\bar{b}\gamma v} - S_w^H - S_w^M \frac{\bar{a}}{\gamma \bar{b}} - S_w^L \frac{\bar{a}}{\gamma v} \right) \frac{\bar{b}\gamma v}{\bar{b}\gamma v + \bar{a}v + \bar{a}\bar{b}} x \\ c &= y \beta \frac{\bar{b}}{\bar{b} + \bar{a} + \bar{a}\bar{b}} \\ d &= - \left[(\alpha + \phi) - \beta \frac{\bar{b}}{\bar{b} + \bar{a} + \bar{a}\bar{b}} x \right] \end{aligned}$$

The stability of the equilibrium point studied by local condition. It is seen from the eigenvalue that generated by substitute the equilibrium point without employment rate and the wages share $T_1(x, y)$ and the existence equilibrium point of employment rate and wages share $T_2(x^*, y^*)$ into the Jacobian matrix $J(x, y)$ in equation (12), the eigen value of the the equilibrium point $T_1(x, y)$ follows:

$$\lambda_1 = S_c \sigma - \delta - \phi - n \quad \vee \quad \lambda_2 = -\alpha - \phi$$

To satisfy the stability, all the eigenvalues must be negative. Thus the stability of equilibrium point of $T_1(x, y)$ is a stable node locally condition if it satisfy $S_c \sigma < \phi + \delta + n$. And the eigen value of the the equilibrium point $T_2(x^*, y^*)$ follows:

$$\lambda_{12} = \pm i \sqrt{(\alpha + \phi)(S_c \sigma - \delta - \phi - n)}$$

From the eigen value obtained, the stability of equilibrium point of $T_2(x^*, y^*)$ center locally condition when $S_c \sigma > \phi + \delta + n$. So that the stability of the equilibrium point without employment rate and the wages share $T_1(x, y)$ and the existence equilibrium point of employment rate and wages share $T_2(x^*, y^*)$ based on the eigenvalues obtained results such as in Table 4.

Table 4. The Equilibrium Stability of the Goodwin Model of The Entire Economy (Capitalists)

The Stability	T_1	T_2
$S_c \sigma > \phi + \delta + n$	Saddle	Local center
$S_c \sigma < \phi + \delta + n$	Local stable node	Saddle

Numerical Simulation of Goodwin Model Modification with Three Types of Workers

Numerical simulations are used to show whether Goodwin's model of three types of workers satisfy the ideal of Goodwin's model cycle. Simulation on Goodwin model with three types of workers conducted by substituting parameter values that have been estimated (using ordinary least squares) and determined based on Indonesian economic data from 2000 to 2020. The data obtained from website of Badan Pusat Statistik Indonesia [16]. Before estimating the parameters, the step that must be done is to find out which conditions in the data from 2000 to 2020 that satisfy the Phillips Curve theory in Goodwin's model which is satisfying the conditions of the inverse relationship between the unemployment rate and the wage growth rate. The results obtained conditions that satisfy the Phillips Curve theory in the Goodwin model are in the period 2001-2004 and the period 2010-2014. From the results obtained, the data we used is data per year divided by two periods, there are period 2001-2004 and period 2010-2014. Furthermore, estimated the parameters in each period. Simulation uses Python applications to generate solution field curves and phase field curves, by providing initial values for each variable. The parameters value of the model is presented on Table 5.

Table 5. Estimation of parameters value

Parameter	Description	Value of Period 2001-2004	Value of Period 2010-2014	Source
α	Parameters that affect wages	1.4305	0.6732	Estimation
β	Parameters that affect wages	13.3076	8.2391	Estimation
σ	Capital Productivity	1.7883	2.8918	Estimation
ϕ	Parameters of increasing labor productivity	0.0991	0.0867	Estimation
n	Population growth rate	0.0137	0.0135	Estimation
$\bar{a}$	Parameters that affect the labor productivity of high-skilled workers	1.3980	1.0929	Estimation
$\bar{b}$	Parameters that affect the labor productivity of medium-skilled workers	0.5255	0.5594	Estimation
γ	Parameters that affect the real wage rate of high-skilled workers	1.6335	1.6606	Estimation
ν	Parameters that affect the real wage rate of medium-skilled workers	1.4058	1.3536	Estimation
S_c	Saving rate of the entire economy (capitalists)	3.9611	3.3436	Estimation
S_w^H	Saving rate of high-skilled workers	0.7627	0.4964	Estimation
S_w^M	Saving rate of medium-skilled workers	0.7134	0.4321	Estimation
S_w^L	Saving rate of low-skilled workers	0.6314	0.4256	Estimation
δ	Capital depreciation rate	$0 < \delta < 1$	$0 < \delta < 1$	[17]

The initial value of Goodwin model of high-skilled workers (x_H and y_H), Goodwin model of medium-skilled workers (x_M and y_M), Goodwin model of low-skilled workers (x_L and y_L), Goodwin model of the entire economy (capitalists) (x and y) are 0.2. The simulation results using parameter values on Table 5 and an initial value of 0.2 are obtained as follows:

Goodwin Model Simulation with Three Types of Workers

Goodwin Model Simulation of High-Skilled Workers Period 2001-2004



Figure 1. The Solutions Field of Goodwin Model of High-Skilled Workers Period 2001-2004

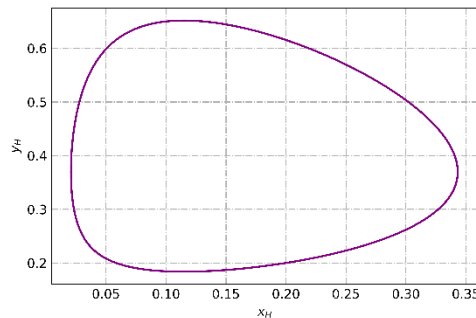


Figure 2. The Phase Field of Goodwin Model of High-Skilled Workers Period 2001-2004

Goodwin Model Simulation of High-Skilled Workers Period 2010-2014



Figure 3. The Solutions Field of Goodwin Model of High-Skilled Workers Period 2010-2014

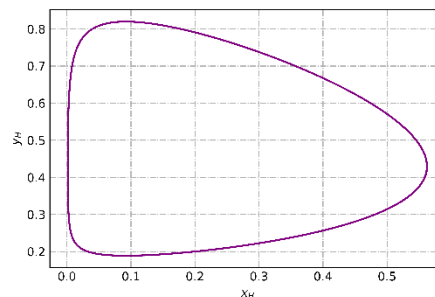


Figure 4. The Phase Field of Goodwin Model of High-Skilled Workers Period 2010-2014

Figures 1 and 2 show the dynamic cycle between employment rate and the wage share of the high-skilled workers period 2001-2004. Figure 1 shows that the employment rate and wage share of the high-skilled workers increase at the beginning of the cycle followed by a decrease towards stability then fluctuate again every 20 years. The trivial equilibrium point $T_1(x_H, y_H) = (0,0)$ is unstable (saddle) with eigenvalues $\lambda_1 = 6.95$ and $\lambda_2 = -1.53$ and $T_2(x_H^*, y_H^*) = (0.11, 0.37)$ is center with eigen values $0. + 3.26i, 0. - 3.26i$ in accordance with the results of stability analysis in Table 1.

Figure 3 and Figure 4 show the dynamic cycle between employment rate and the wage share of the high-skilled workers period 2010-2014. The result of the cycle is similar to the period 2001-2004 but the fluctuations occur every 30 years. This shows that in the period 2010-2014 the fluctuations were relatively slower than the period 2001-2004. Fluctuations result from both periods occur periodically, this indicate that

the stability of the fixed point is the center as shown in the phase fields of Figure 2 and Figure 4. The trivial equilibrium point $T_1(x_H, y_H) = (0,0)$ is unstable (saddle) with eigenvalues $\lambda_1 = 9.55$ and $\lambda_2 = -0.76$ and $T_2(x_H^*, y_H^*) = (0.09, 0.43)$ is center with eigen values $8.88 \times 10^{-16} + 2.7i, 8.88 \times 10^{-16} - 2.7i$ in accordance with the results of stability analysis in Table 1.

Goodwin Model Simulation of Medium-Skilled Workers Period 2001-2004

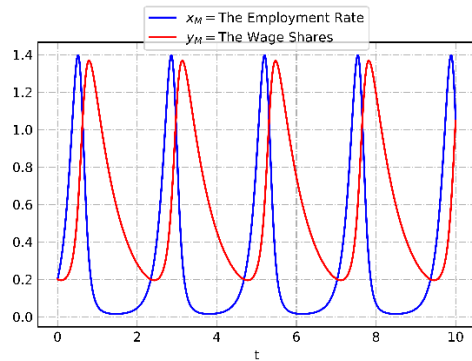


Figure 5. The Solutions Field of Goodwin Model of Medium-Skilled Workers Period 2001-2004

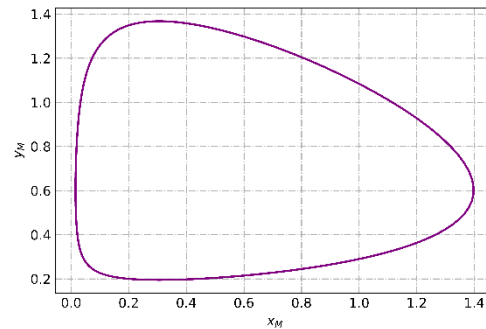


Figure 6. The Phase Field of Goodwin Model of Medium-Skilled Workers Period 2001-2004

Goodwin Model Simulation of Medium-Skilled Workers Period 2010-2014

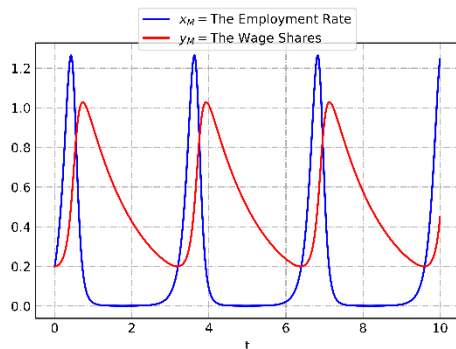


Figure 7. The Solutions Field of Goodwin Model of Medium-Skilled Workers Period 2010-2014

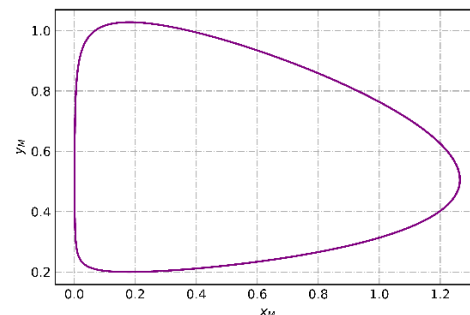


Figure 8. The Phase Field of Goodwin Model of Medium-Skilled Workers Period 2010-2014

Figures 5 and 6 show the dynamic cycle between employment rate and the wage share of the medium-skilled workers period 2001-2004. Figure 5 shows that the employment rate decreases at the beginning of the cycle followed by a rise and fluctuates again every 25 years. While the wage share begins with an increase and then decreases to stability and fluctuates again every 25 years. The trivial equilibrium point $T_1(x_M, y_M) = (0,0)$ is unstable (saddle) with eigenvalues $\lambda_1 = -1.53$ and $\lambda_2 = 6.95$ and $T_2(x_M^*, y_M^*) = (0.30, 0.60)$ is center with eigen values $8.88 \times 10^{-16} + 2.7i, 8.88 \times 10^{-16} - 2.7i$ in accordance with the results of stability analysis in Table 2.

Figure 7 and Figure 8 show the cycle of dynamics between employment rate and the wage share of the medium-skilled workers period 2010-2014. The result of the cycle is similar to the period 2001-2004 but the fluctuations occur every 30 years. This shows that in the period 2010-2014 the fluctuations were relatively slower than the period 2001-2004. Fluctuations result from both periods occur periodically, this indicate that

the stability of the fixed point is center as shown in the phase fields of Figure 6 and Figure 8. The trivial equilibrium point $T_1(x_M, y_M) = (0,0)$ is unstable (saddle) with eigenvalues $\lambda_1 = -0.75$ and $\lambda_2 = 9.55$ and $T_2(x_M^*, y_M^*) = (0.18, 0.50)$ is center with eigen values $8.88 \times 10^{-16} + 2.7i, 8.88 \times 10^{-16} - 2.7i$ in accordance with the results of stability analysis in Table 2.

Goodwin Model Simulation of Low-Skilled Workers Period 2001-2004

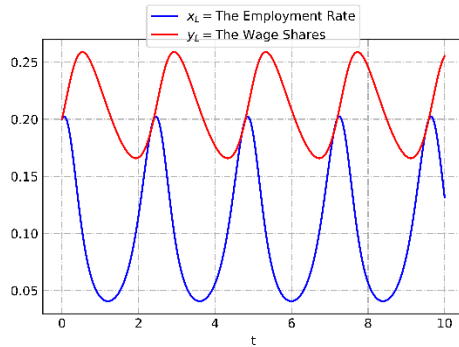


Figure 9. The Solutions Field of Goodwin Model of Low-Skilled Workers Period 2001-2004

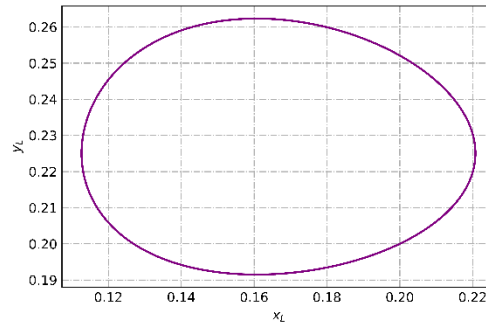


Figure 10. The Phase Field of Goodwin Model of Low-Skilled Workers Period 2001-2004

Goodwin Model Simulation of Low-Skilled Workers Period 2010-2014

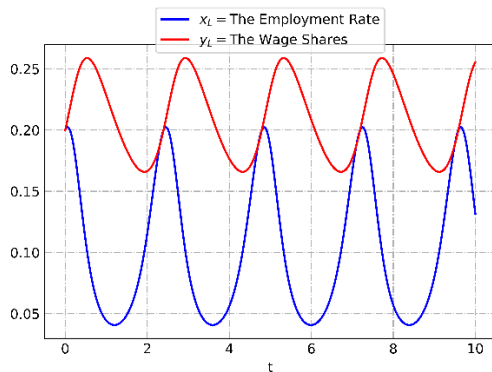


Figure 11. The Solutions Field of Goodwin Model of Low-Skilled Workers Period 2010-2014

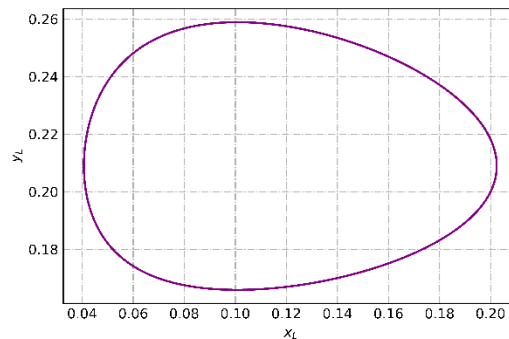


Figure 12. The Phase Field of Goodwin Model of Low-Skilled Workers Period 2010-2014

Figures 9 and 10 show the dynamic cycle between employment rate and the wage share of the low-skilled workers period 2001-2004. Figure 9 shows that employment rate and the wage share of the low-skilled workers increase at the beginning of the cycle followed by a decrease to stability then fluctuates again every 20 years. The trivial equilibrium point $T_1(x_L, y_L) = (0,0)$ is unstable (saddle) with eigenvalues $\lambda_1 = 6.95$ and $\lambda_2 = -1.53$ and $T_2(x_L^*, y_L^*) = (0.16, 0.22)$ is center with eigen values $0. + 3.26i, 0. - 3.26i$ in accordance with the results of stability analysis in Table 3.

Figure 11 and Figure 12 show the cycle of dynamics between employment rate and the wage share of the low-skilled workers period 2010-2014. The resulting cycle is similar to the period 2001-2004 but the fluctuations occur every 25 years. This shows

that in the period 2010-2014 the fluctuations were relatively slower than the period 2001-2004. Fluctuations result from both periods occur periodically, this indicate that the stability of the fixed point is the center as shown in the phase fields of Figure 10 and Figure 12. The trivial equilibrium point $T_1(x_L, y_L) = (0,0)$ is unstable (saddle) with eigenvalues $\lambda_1 = 9.56$ and $\lambda_2 = -0.76$ and $T_2(x_L^*, y_L^*) = (0.10, 0.20)$ is center with eigen values $8.88 \times 10^{-16} + 2.7i, 8.88 \times 10^{-16} - 2.7i$ in accordance with the results of stability analysis in Table 3.

Goodwin Model Simulation of the Entire Economy (Capitalists) Period 2001-2004

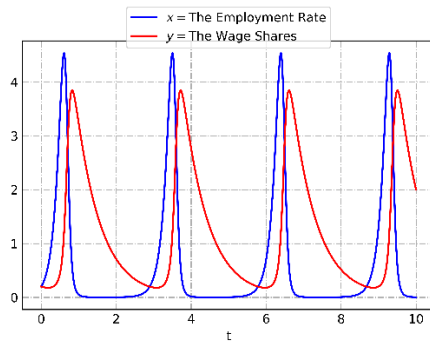


Figure 13. The Solutions Field of Goodwin Model of The Entire Economy (Capitalists) Period 2001-2004

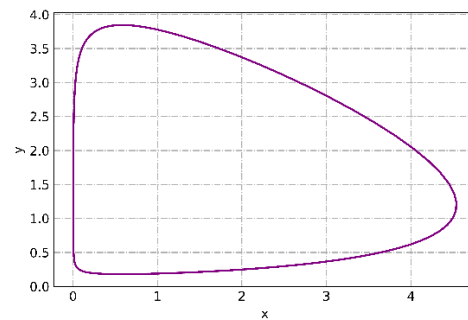


Figure 14. The Phase Field of Goodwin Model of The Entire Economy (Capitalists) Period 2001-2004

Goodwin Model Simulation of the Entire Economy (Capitalists) Period 2010-2014

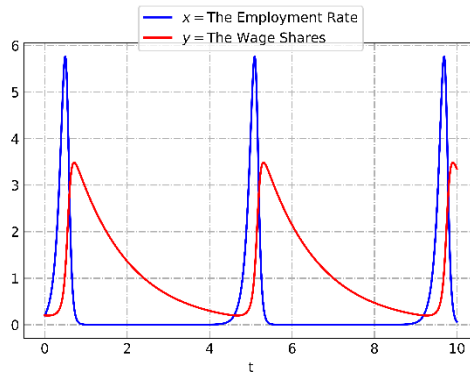


Figure 15. The Solutions Field of Goodwin Model of The Entire Economy (Capitalists) Period 2010-2014

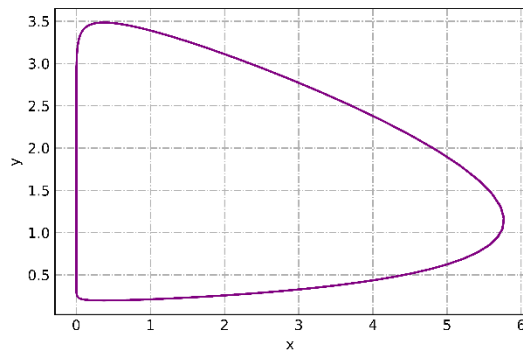


Figure 16. The Phase Field of Goodwin Model of The Entire Economy (Capitalists) Period 2010-2014

Figures 13 and 14 show the cycle of dynamics between the employment rate and the wage share of the entire economy (capitalist) period 2001-2004. Figure 13 shows that the employment rate decreases at the beginning of the cycle followed by a rise and fluctuates again every 25 years. While the wage share begins with an increase and then decreases to stability and fluctuates again every 25 years. The trivial equilibrium point $T_1(x, y) = (0,0)$ is unstable (saddle) with eigenvalues $\lambda_1 = 6.95$ and $\lambda_2 = -1.53$ and $T_2(x^*, y^*) = (0.58, 1.20)$ is center with eigen values $0. + 3.26i, 0. - 3.26i$ in accordance with the results of stability analysis in Table 3.

Figure 15 and Figure 16 show the cycle of dynamics between employment rate and

the wage share of the entire economy (capitalists) period 2010-2014. The result of the cycle is similar to the period 2001-2004 but the fluctuations occur every 45 years. This shows that in the period 2010-2014 the fluctuations were relatively slower than the period 2001-2004. Fluctuations result from both periods occur periodically, this indicate that the stability of the fixed point is the center as shown in the phase fields of Figure 14 and Figure 16. The trivial equilibrium point $T_1(x, y) = (0, 0)$ is unstable (saddle) with eigenvalues $\lambda_1 = 9.56$ and $\lambda_2 = -0.76$ and $T_2(x^*, y^*) = (0.37, 1.14)$ is center with eigenvalues $0. + 2.7i, 0. - 2.7i$ in accordance with the results of stability analysis in Table 3.

Numerical simulation results of the period 2001-2004 and the period 2010-2014 showed that the cycle produced by each worker in the period 2010-2014 fluctuated relatively slower than the period 2001-2004. This shows that in the period 2001-2004 the resulting cycle is more ideal because the recovery process to reach the peak (expansion) of the economy tends to be faster than in the period 2010-2014. Low-skilled workers produce more ideal cycles than high-skilled, medium- workers and the entire economy (capitalists) because low-skilled workers have a relatively rapid period of fluctuations, with 2001-2004 fluctuations occurring every 20 years and in the period 2010-2014 of fluctuations occur every 25 years. So it can be concluded that low-skilled workers in the period 2001-2004 had an ideal cycle with the recovery process to reach the peak (expansion) of the economy occurred every 20 years.

The Effect of Reducing the Wage Gap Between Three Types of Workers

In this section, an investigation into the effects of reducing the wage gap between the three types of workers by reducing the value of wage gap parameters between high- and medium-skilled workers (γ) and the parameters of the wage gap between medium- and low-skilled workers (v) at each period by 10%, 20% and 30%. So that the parameter values γ in the period 2001-2004 from $\gamma = 1.633548656$ to $\gamma_1 = 1.47019379$, $\gamma_2 = 1.306838925$, $\gamma_3 = 1.143484059$ and in the period 2010-2014 from $\gamma = 1.660567744$ to $\gamma_1 = 1.49451097$, $\gamma_2 = 1.328454195$, $\gamma_3 = 1.162397421$. Parameter v in the period 2001-2004 from $v = 1.405799381$ to $v_1 = 1.265219443$, $v_2 = 1.124639505$, $v_3 = 0.984059567$ and in the period 2010-2014 from $v = 1.353648378$ to $v_1 = 1.21828354$, $v_2 = 1.082918702$, $v_3 = 0.947553865$. The simulation results are presented in Figure 17 through Figure 24.

Goodwin Model Numerical Simulation with Three Types of Workers When Reducing the Value of Parameters γ and v

Goodwin Model of High-Skilled Workers with Reduced The Value of Parameters γ and v

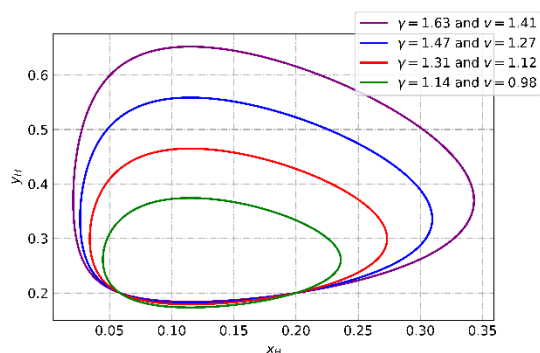


Figure 17. The Sensitivity of Parameter γ and v of High-Skilled Workers Period 2001-2004

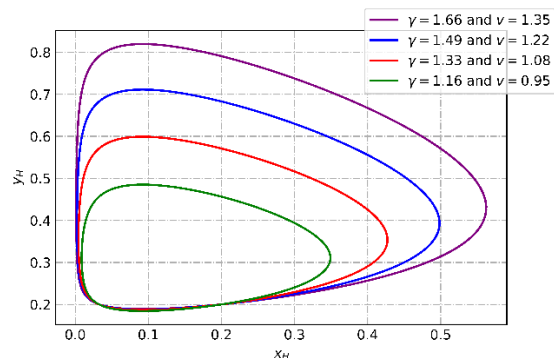


Figure 18. The Sensitivity of Parameter γ and v of High-Skilled Workers Period 2010-2014

Figure 17 and Figure 18 show the cycle of dynamics between employment rate and the wage share of the high-skilled workers with a reduction in the wage gap between the three types of workers. The results showed the greater the decrease in the value of the γ and ν the more shifting the orbit downwards and minimized the cycle. This indicates that the smaller value of the γ and ν then the economic cycle becomes unstable.

Goodwin Model of Medium-Skilled Workers with Reduced The Value of Parameters γ and ν

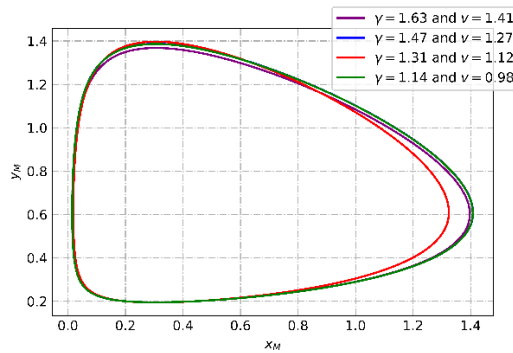


Figure 19. The Sensitivity of Parameter γ and ν of Medium-Skilled Workers Period 2001-2004

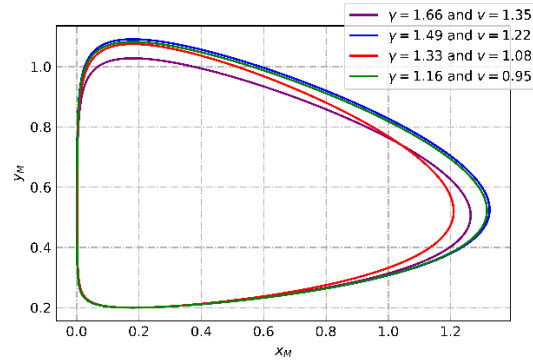


Figure 20. The Sensitivity of Parameter γ and ν of Medium-Skilled Workers Period 2010-2014

Figure 19 and Figure 20 show the cycle of dynamics between employment rate and the wage share of the medium-skilled workers with a reduction in the wage gap between the three types of workers. Figure 19 shows the reduction value of γ and ν as 10% and 30% maximizing the cycle and a 20% reduction minimizing the cycle but not shifting the orbit. This indicates that the smaller the value of the γ and ν then the economic cycle formed is not fixed but the cycle formed is stable because it does not cause significant cyclical changes from the previous model. Figure 20 shows the reduction value of γ and ν as 10% and 20% maximizing the cycle and a 30% reduction minimizing the cycle but not shifting the orbit. This indicates that the smaller the value of the γ and ν then the economic cycle formed is also not fixed but the cycle formed is stable because it does not cause significant cyclical changes from the previous model. Of the two cycles formed, the more ideal cycle is in the period 2001-2004 because it produces a cycle change that is not significant compared to the period 2010-2014.

Goodwin Model of Low-Skilled Workers with Reduced The Value of Parameters γ and ν

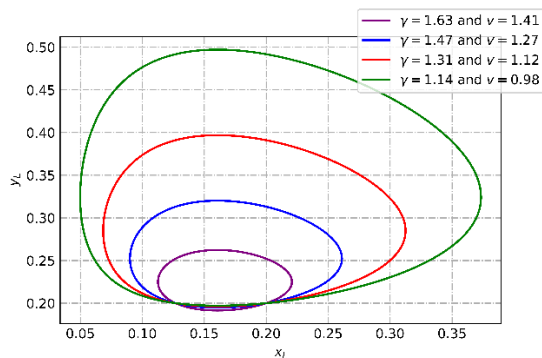


Figure 21. The Sensitivity of Parameter γ and ν of Low-Skilled Workers Period 2001-2004

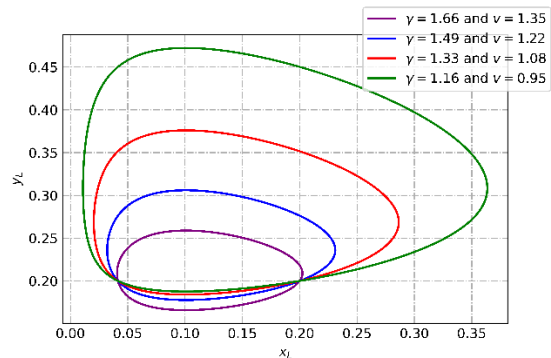


Figure 22. The Sensitivity of Parameter γ and ν of Low-Skilled Workers Period 2010-2014

Figure 21 and Figure 22 show the cycle of dynamics between employment rate and the wage share of the low-skilled workers with a reduction in the wage gap between the three types of workers. Figure 21 and Figure 22 show the greater the decrease in the value of the γ and ν further shifting the orbit upwards and enlarging the cycle. This indicates that the reduction of the value of γ and ν makes good changes characterized by an enlarged cycle, that indicates a large economic recovery process but cannot stabilize the economy due to very significant differences with the previous model and hence the economic cycle becomes unstable.

Goodwin Model of The Entire Economy (Capitalists) with Reduced The Value of Parameters γ and ν

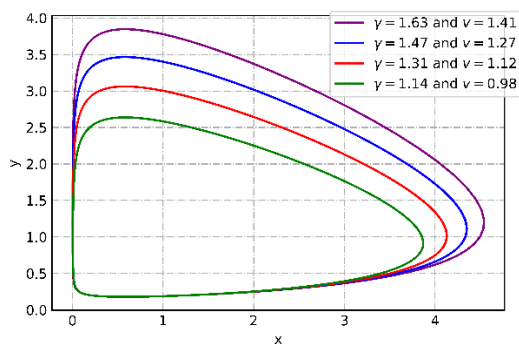


Figure 23. The Sensitivity of Parameter γ and ν of The Entire Economy (Capitalists) Period 2001-2004

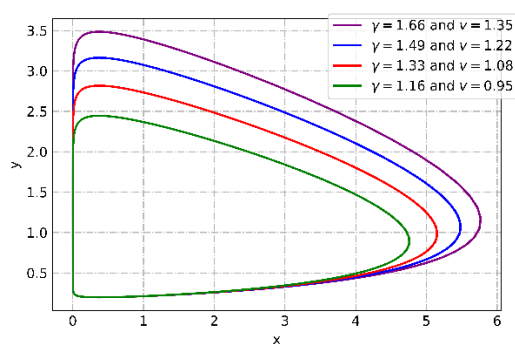


Figure 24. The Sensitivity of Parameter γ and ν of The Entire Economy (Capitalists) Period 2010-2014

Figure 23 and Figure 24 show the cycle of dynamics between employment rate and the wage share of the entire economy (capitalists) with a reduction in the wage gap between the three types of workers. Figure 23 and Figure 24 show the greater decrease the value of the γ and ν minimizing the cycle. This indicates that the smaller the value of the γ and ν then the economic cycle becomes unstable.

Numerical simulation results of the period 2001-2004 and the period 2010-2014 on each type of worker's skill showed when the values of γ and ν decreased by 10%, 20%, and 30% resulting in an unstable economic cycle in high- and low-skilled workers and the entire (capitalist) economy. The reduction in the value of γ and ν appears to be desired by low-skilled workers but not for economic stability because the resulting cycle shifts the orbit upwards and enlarges the cycle which means this reduction in gap makes good changes characterized by the grinding of the orbit upwards and the enlarging of the cycle. While in medium-skilled workers the resulting cycle stabilizes the economy. So it can be concluded that reducing the wage gap cannot stabilize the economy as a whole. The wage gap must be determined according to each worker's ability to produce a stable economic cycle.

CONCLUSIONS

Goodwin's model is constructed based on economic variables that can describe the dynamics between employment opportunity rates and wages share. Goodwin's model is modified by adding consideration of the type of workers from two types of workers,

namely high and low-skilled workers into three types of workers, namely high, medium and low-skilled workers. The results showed two equilibrium points, namely The Equilibrium Point without Employment Rate and The Wages Share (T_1) and the Existence Equilibrium Point of Employment Rate and Wages Share (T_2). T_1 achieves a stable node condition when $S_c\sigma < \delta + \phi + n$ while T_2 reaches a stable center condition when $S_c\sigma > \delta + \phi + n$. Based on Indonesia's economic data from 2000 to 2020, conditions were obtained that satisfy the Phillips curve theory in the Goodwin Model are the period 2001-2004 and the period 2010-2014. Numerical simulations showed that Goodwin's model with three types of workers fluctuated periodically. From the estimation results, low-skilled workers produce ideal Goodwin model cycle while Goodwin's model of high-, medium-skilled workers and the entire economy (capitalist) does not produce ideal Goodwin model cycle. Furthermore, gap reduction is carried out to see the effect on each type of worker's skills in economic stability. Numerical simulation results show when the values of γ and ν decrease by 10%, 20%, and 30% resulting in an unstable economic cycle in high-skilled, low-skilled workers and the entire economy (capitalists). Reductions in the value of γ and ν appear to be desirable by low-skilled workers but not for economic stability. And for medium-skilled workers, the resulting cycle can stabilize the economy. Thus, the wage gap cannot stabilize the overall economy and must be determined according to each worker's ability to produce a stable economic cycle.

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