DISSIPATIVE ANALYSIS OF CONTINUOUS-TIME SYSTEMS WITH TWO ADDITIVE TIME-VARYING DELAYS

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ABSTRACT

This paper presents an application of dissipative concept for stability analysis of continuous-time system with two additive time-varying delays in the state. Our attention is focused on analysis of whether the continuous-time system with two additive time-varying delays in the state is asymptotically stable and dissipative. By exploiting Lyapunov-Krasovski functional and introducing free weighting matrix variables, the stability condition is derived by using linear matrix inequality (LMI) techniques.

Keywords: *Dissipative analysis, linear matrix inequality (LMI), time delay systems, networked control systems*

1. INTRODUCTION

The study of dissipative concept which is used to analyze and design of control systems was initially developed by Willems [1]. A system has the dissipative property if it always dissipates the energy. Many physical systems have input-output properties related to the conservation, dissipation and transport energy. Willems in [1] stated the following definition.

Definition 1. System $\dot{x} = f(x,w)$, $z = g(x,w)$, where x is the system state, w represents input to the system, z is the system output, is dissipative with respect to the supply function s(w, z), if there exists a storage function, $V(x) \geq 0$, such that the dissipation inequality

$$V(x(t_1)) \leq V(x(t_0)) + \int_{t_0}^{t_1} s(w(t), z(t))dt \tag{1}$$

holds along all possible trajectories of the system and for all $t_0 \leq t_1$.

In the differential form, (1) is equivalent to the differential dissipation inequality,

$$\dot{V}(x) \leq s(w(t), z(t)) \tag{2}$$

Equation (2) states that the rate of change of the stored energy is less than or equal to the input power, the difference being the rate of the energy dissipation. Willems in [2] showed that the notion of a dissipative system is a natural generalization of a Lyapunov functon. Analysis of dissipativity is a problem of finding a storage function (as a Lyapunov function candidate) which satisfy (2) with respect to a certain supply rate. In general, supply function is stated in quadratic function defined by

$$\begin{aligned} s(w(t), z(t)) &= \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} \\ &= z^T(t)Qz(t) + w^T(t)S^T z(t) + z^T(t)Sw(t) + w^T(t)Rw(t) \end{aligned} \tag{3}$$

Here, Q, S and R are real matrices with appropriate dimension and Q and R are symmetric matrices. The dissipative inequality (2) with supply function (3) is quite general, covering bounded real lemma and positive real (or passivity-based) as special cases.

Remark 1. Dissipativity analysis with quadratic supply function is quite general which includes H_∞ and passivity as special cases.

(1) H_∞ performance analysis can be covered by supply function (3) when $Q = -I$, $S = 0$ and $R = \gamma^2 I$.

(2) Passive systems are dissipative with respect to supply function (3) with $R = Q = 0$ and $S = I$.

Without loss of generality, we shall make the following assumption.